Generalized concept ----

"Angle modulation"
"Exponential modulation"

Recall ...

$$\phi_{PM} = A\cos(\omega_c t + k_p m(t))$$

$$\phi_{FM} = A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\,d\alpha\right)$$

We can generalize this as:

$$\phi_{EM} = A\cos\left[\omega_c t + \underbrace{\int_{-\infty}^{t} m(\alpha)\,h(t-\alpha)\,d\alpha}_{\text{Convolution}}\right]$$

$$h(t) = k_p\,\delta(t) \rightarrow PM$$
$$h(t) = k_f\,u(t) \rightarrow FM$$

Bandwidth $\approx 2k_f m_p$ — m_p = peak amplitude of $m(t)$

or $2k_p m'_p$ — m'_p = peak amplitude of $\dot{m}(t)$

Origin of this guess of bandwidth ...
concept of instantaneous frequency

PM: $\omega_i(t) = \omega_c + k_p \dot{m}(t)$

FM: $\omega_i(t) = \omega_c + k_f m(t)$

just find the min and max instantaneous frequency.

→ This is just an approximation.
It's really worse than this

→ Depends on modulation level,
not modulation frequency

→ Actually – it's infinite, use "essential bandwidth".

HW:
5.1 – 1, 2, 3

Bandwidth (not just a guess)

Define (FM in exponential form)

$$\hat{\phi}_{FM}(t) = A\,e^{j[\omega_c t + k_f a(t)]} = A\,\underbrace{e^{jk_f a(t)}}_{\text{modulation}}\,\underbrace{e^{j\omega_c t}}_{\text{carrier}}$$

where ...

$$a(t) = \int_{-\infty}^{t} m(\alpha)\,d\alpha$$

$$\phi_{FM}(t) = \mathrm{Re}\left(\hat{\phi}_{FM}(t)\right)$$ ← to relate to the way we did it last time.

Expand $e^{jk_f a(t)}$ in power series ---

$$\hat{\phi}_{FM}(t) = A\underbrace{\left[1 + jk_f a(t) - \frac{(k_f a(t))^2}{2!} + \cdots + j^n \frac{(k_f a(t))^n}{n!} + \cdots\right]}_{\text{the side bands. — lots of them.}} e^{j\omega_c t}$$

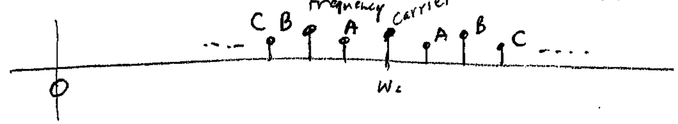
$$\phi_{FM}(t) = \mathrm{Re}\left(\hat{\phi}_{FM}(t)\right) =$$

$$= A\Big[\underbrace{\cos(\omega_c t)}_{\text{unmodulated Carrier}} \underbrace{- k_f a(t)\sin(\omega_c t) - \frac{(k_f a(t))^2}{2!}\cos(\omega_c t) + \frac{(k_f a(t))^3}{3!}\sin(\omega_c t) - \cdots}_{\text{AM side bands ?}}\Big]$$

Ⓐ fundamental modulating frequency

Ⓑ 2nd harmonic

Ⓒ 3rd harmonic



etc....

$a(t)$ is the result of a linear operation on $m(t)$ — so has same bandwidth.

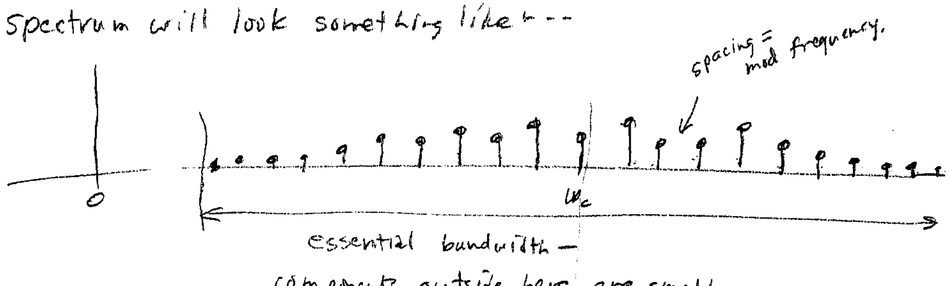
Signal	Bandwidth
$a(t)$	B
$a^2(t)$	$2B$
$a^3(t)$	$3B$
$\vdots$	
$a^n(t)$	nB

→ FM has multiple sidebands — infinite bandwidth
(but "essential" bandwidth is finite)

Suppose $m(t) = \cos(2000\pi t)$ (1 kHz)

and peak deviation = 10 kHz ..

spectrum will look something like ---



Suppose $m(t) = \cos(10000\, t)$ (5 kHz) —