

Chapter 2 — Review (?) of Signals

IB-1

- Basic concepts, definitions (today) (2.1, 2.2, 2.3, 2.4)
- "Correlation" — finding whether signals (Thursday?) (2.5, 2.6) are the same after being altered by a channel
- Fourier Series — Correspondence between frequency + time domain (Friday?) (2.7, 2.8, 2.9)

Review (?) of Signals (chapter 2)

IB-2

2.1 "Size" of a signal --
Energy + Power

$$E = \int_{-\infty}^{\infty} g^2(t) dt$$

↑
watts-seconds
joules

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(t) dt$$

↑
watts = average energy

but we use a more abstract version here.

Consider -- (DC) $g(t) = 1$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} (1)^2 dt = 1 \quad \leftarrow \text{doesn't accumulate in time}$$

$$E = \int_{-\infty}^{\infty} (1)^2 dt = \infty$$

$$g(t) = \begin{cases} 1 & 0 \leq t \leq 10 \\ 0 & t < 0 \\ 0 & t > 10 \end{cases}$$

$$E = \int_{-\infty}^{\infty} 1 dt = \int_0^{10} 1 dt = 10 \quad \leftarrow \text{accumulates in time.}$$

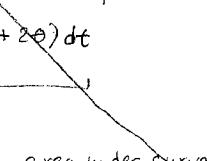
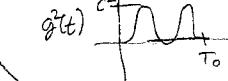
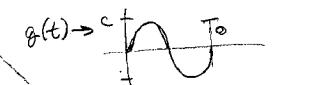
$$g(t) = C \cos(\omega_0 t + \theta)$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} C^2 \cos^2(\omega_0 t + \theta) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{C^2}{2} [1 + \cos(2\omega_0 t + 2\theta)] dt$$

$$= \lim_{T \rightarrow \infty} \frac{C^2}{2T} \int_{-T/2}^{T/2} dt + \lim_{T \rightarrow \infty} \frac{C^2}{2T} \int_{-T/2}^{T/2} \cos(2\omega_0 t + 2\theta) dt$$

$$= \frac{C^2}{2} \quad \leftarrow \text{confirms } P = \frac{(V_{\text{peak}})^2}{2}$$



area under curve

Power

$$g(t) = 1$$

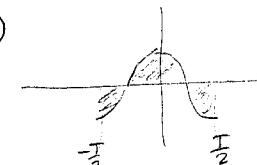
$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (1)^2 dt = \frac{1}{T} \left[t \right]_{-\frac{T}{2}}^{\frac{T}{2}} = 1$$

IB-3

$$g(t) = 2$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (2)^2 dt = \frac{1}{T} \left[2t \right]_{-\frac{T}{2}}^{\frac{T}{2}} = 4$$

confirms $P = V^2$



$$g(t) = C \cos(\omega_0 t + \theta)$$

$$\begin{aligned} P &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} (C \cos(\omega_0 t + \theta))^2 dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} C^2 \cos^2(\omega_0 t + \theta) dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{C^2}{2} [1 + \cos(2\omega_0 t + 2\theta)] dt \\ &= \underbrace{\lim_{T \rightarrow \infty} \frac{C^2}{2T} \int_{-\frac{T}{2}}^{\frac{T}{2}} 1 dt}_{=1} + \underbrace{\lim_{T \rightarrow \infty} \frac{C^2}{2T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\omega_0 t + 2\theta) dt}_{=0} \\ &= \frac{C^2}{2} \end{aligned}$$

confirms $P = \frac{(V_{\text{peak}})^2}{2} = (V_{\text{rms}})^2$

IB-4

Continuous time vs. discrete time
(samples)

Analog vs. digital
(continuous magnitude) (discrete magnitudes)

Periodic vs. aperiodic

(repeats)

$$g(t) = g(t + T_0)$$

for all t

→ For analysis —
often we consider aperiodic
signals to be periodic,
by repeating them.

Energy vs. power signals

Energy signals → finite energy
Finite duration gives finite energy.

Power signals → finite, non zero, power.

Must have infinite duration, which gives infinite energy.

Truly periodic signals are power signals

Deterministic vs. random signals.



Known completely

Can be expressed
completely in
mathematical
or graphical form

Known only in a probabilistic sense

Most noise is random.
(but not all)

All useful message signals
are random.

(If it is deterministic,
why bother sending it?)

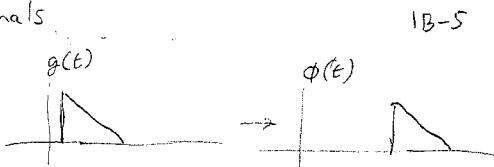
2-3 - Operations on signals

Time shifting

$$\phi(t+T) = g(t)$$

or $\phi(t) = g(t-T)$

T = delay



Time shifting in a channel
is inevitable.

IB-5

IB-6

Time inversion (Time reversal)



$$\phi(-t) = g(t)$$

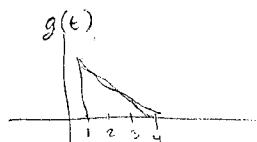
or $\phi(t) = g(-t)$

playing a signal backwards.

Time Scaling

Expansion or compression in time

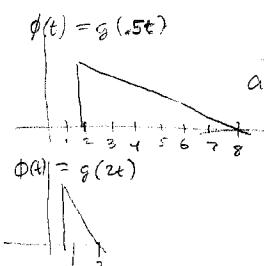
$$\phi(t) = g(at)$$



$a < 1 \rightarrow$ expanded

in time
(slowed down)

$a > 1 \rightarrow$ compressed
in time
(speeded up)



Expansion in time reduces bandwidth

Compression in time increases bandwidth

Channels usually don't do this,

but we might do it deliberately.

2.4 Unit impulse function

Defined as $\delta(t) = 0 \quad t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad \textcircled{1}$$

(implies it must be ∞ when $t=0$)

Practically - could think of it as a rectangular pulse, width = ϵ , $\epsilon \rightarrow 0$

Multiplication of function by impulse

$$\phi(t) \delta(t) = \phi(0) \delta(t) \quad \textcircled{2}$$

↑ only the value at $t=0$ matters.

With time shifting -

$$\phi(t) \delta(t-T) = \phi(T) \delta(t-T) \quad \textcircled{3}$$

↑ only the value at $t=T$ matters.

$$\begin{aligned} \text{Sampling - } \int_{-\infty}^{\infty} \phi(t) \delta(t) dt &= \int_{-\infty}^{\infty} \phi(0) \delta(t) dt \quad \text{from } \textcircled{2} \\ &= \phi(0) \int_{-\infty}^{\infty} \delta(t) dt \\ &= \phi(0) \quad \text{from } \textcircled{1} \end{aligned}$$

→ Area under the product of a function with an impulse equals the value of the function at that instant

$$\int_{-\infty}^{\infty} \phi(t) \delta(t-T) dt = \phi(T) \quad \text{from } \textcircled{3}$$

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Unit step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



IB-8

A signal is causal if it does not start before $t=0$

e^{-at} is non-causal

$e^{-at} u(t)$ is causal

$$\int_{-\infty}^t \delta(\tau) d\tau = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

integrating an impulse function gives us a unit step.

$$\frac{du}{dt} = \delta(t)$$

differentiating a unit step gives us an impulse.