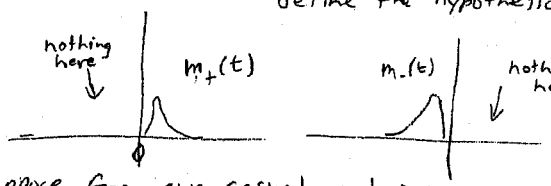
Single sideband — deeper math

Hilbert transform: — Math version of the 90° phase shift.

To derive .... define the hypothetical signals: $m_+(t)$ and $m_-(t)$ ← Complex



Suppose, from our casual analysis: ...

$$m_+(t) = \frac{1}{2}\left[m(t) + j\,m_h(t)\right] \quad (1) \qquad m_-(t) = \frac{1}{2}\left[m(t) - j\,m_h(t)\right] \quad (2)$$

What is $m_h(t)$? (Some books say $\hat{m}(t)$)

Observe: $M_+(\omega) = M(\omega)\,u(\omega)$ (3)

$= \frac{1}{2} M(\omega)\left[1 + \mathrm{sgn}(\omega)\right]$ (4) another way to write it

$= \frac{1}{2}\left[M(\omega) + M(\omega)\,\mathrm{sgn}(\omega)\right]$ (5)

Looking at (1) and (5) ...

Apparently

$$j\,m_h(t) \Leftrightarrow M(\omega)\,\mathrm{sgn}(\omega) \quad (6)$$

divide by j

$$m_h(t) \Leftrightarrow -j\,M(\omega)\,\mathrm{sgn}(\omega) \quad (7)$$

so .... $M_h(\omega) = -j\,M(\omega)\,\mathrm{sgn}(\omega)$ (8)

HW:
4.5-1, 2, 3, 5, 6

F.T. #12 is $\mathrm{sgn}(t) \Leftrightarrow \frac{2}{j\omega}$

by duality

Symmetry property: $\frac{2}{jt} \Leftrightarrow 2\pi\,\mathrm{sgn}(-\omega)$ (9)

divide by $2\pi j$

$$\frac{\cancel{2}}{\cancel{2\pi j}\,t} \Leftrightarrow -j\,\mathrm{sgn}(-\omega) \quad (10)$$

simplify

$$+\frac{1}{\pi t} \Leftrightarrow +j\,\mathrm{sgn}(-\omega) \quad (11)$$

move sign

$$\frac{1}{\pi t} \Leftrightarrow -j\,\mathrm{sgn}(\omega) \quad (12)$$

Apply (12) to (7)

$$\underbrace{m(t) * \frac{1}{\pi t}}_{\text{Corresponds to convolution in time domain}} \Leftrightarrow \underbrace{\overbrace{\left(M(\omega)\right)\left(-j\,\mathrm{sgn}(\omega)\right)}^{\text{factor it}}}_{\text{Multiplication in freq domain}}$$

do convolution gives us:

$$m_h(t) = \underbrace{\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha}\,d\alpha}_{\text{This is "Hilbert transform" of } m(t)}$$

Pass $m(t)$ through a filter with $H(\omega) = -j\,\mathrm{sgn}(\omega)$

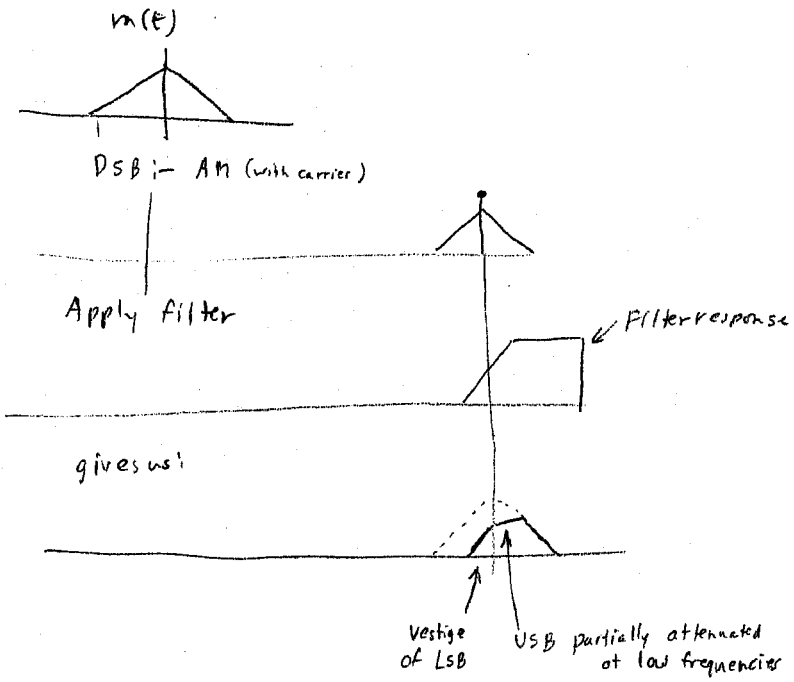
the output will be $m_h(t)$.

$$H(\omega) = \begin{cases} -j = e^{-j\frac{\pi}{2}} & \omega > 0 \\ j = e^{+j\frac{\pi}{2}} & \omega < 0 \\ 0 & \omega = 0 \end{cases}$$

Vestigal sideband

For signals needing good phase response, and DC -- can't use single sideband.

Allow a vestige of the other sideband to pass,
use a filter that allows some overlap!



Then use envelope detector —
with careful filter design,
the vestige adds to the portion of the desired sideband, giving proper response.

Sometimes equalization is used — see Example 4.8 in text,

Remaining AM topics:

Carrier acquisition. (how to synchronize the BFO to the suppressed carrier)

Television

Test Thursday