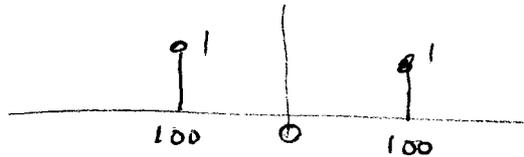


4.5-1 Given  $m(t)$

- i Sketch spectrum of  $m(t)$
- ii Sketch spectrum of the DSB-SC signal  $2m(t) \cos 1000t$
- iii Suppress the LSB spectrum to obtain a USB spectrum
- iv Write  $\phi_{USB}(t)$  for the USB signal
- v Repeat for LSB

(a)  $m(t) = \cos(100t)$

i - spectrum of  $m(t)$



ii DSB-SC  
 $2m(t) \cos(1000t)$



iii USB spectrum



iv. write  $\phi_{USB}(t) = \cos((\omega_c + \omega_m)t)$   
 $= \cos(1100t)$

LSB spectrum

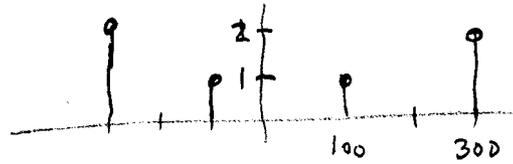


$$\phi_{LSB} = \cos(900t)$$

4.5-1

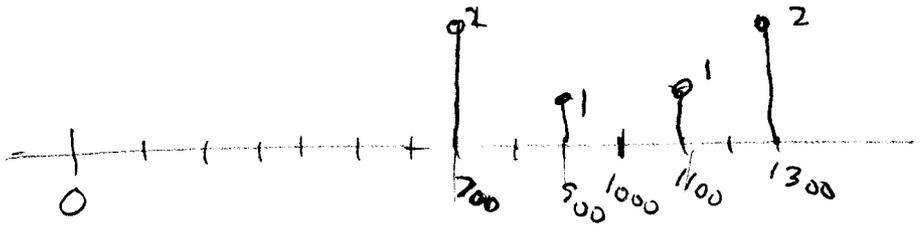
(b)  $m(t) = \cos(100t) + 2\cos(300t)$

i spectrum of  $m(t)$



ii DSC-SC

$2m(t)\cos(1000t)$

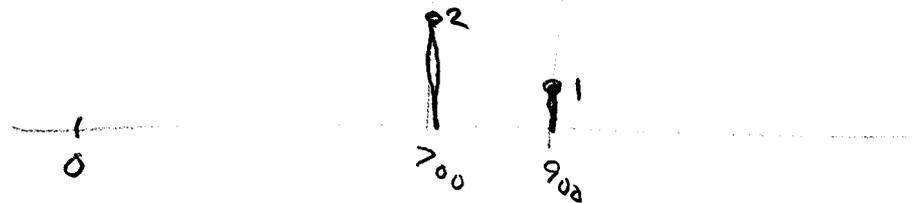


iii USB spectrum



$\phi_{USB} = \cos(1100t) + 2\cos(1300t)$

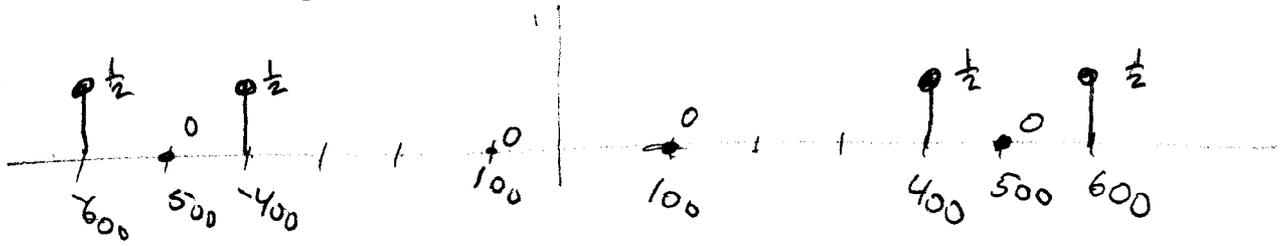
LSB spectrum



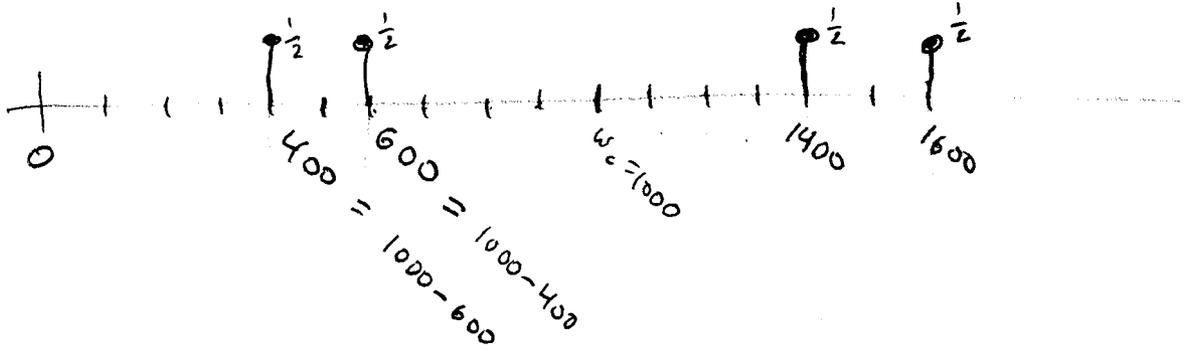
$\phi_{LSB} = \cos(900t) + 2\cos(700t)$

4.5-1 (c)  $m(t) = \cos(100t)\cos(500t)$   
 $= \frac{1}{2}(\cos(400t) + \cos(600t))$

$m(t)$ :



ii DSB-SC

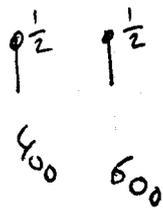


iii USB:



$$\Phi_{USB} = \frac{1}{2}[\cos(1400t) + \cos(1600t)]$$

LSB:



$$\Phi_{LSB} = \frac{1}{2}[\cos(600t) + \cos(400t)]$$

This corresponds to 400t  
 in  $m(t)$

This corresponds to 600t  
 in  $m(t)$

4.5-2

For the same signals, use 4.17 to determine  $\phi_{LSB}$ ,  $\phi_{USB}$   
for  $\omega_c = 1000$

$$4.17 \text{ is: (a) } \phi_{USB}(t) = m(t) \cos(\omega_c t) - m_h(t) \sin(\omega_c t)$$

$$(b) \phi_{LSB}(t) = m(t) \cos(\omega_c t) + m_h(t) \sin(\omega_c t)$$

$$(c) \phi_{SSB}(t) = m(t) \cos(\omega_c t) \mp m_h(t) \sin(\omega_c t)$$

$$(a) \quad m(t) = \cos(100t)$$

$$m_h(t) = \sin(100t)$$

$$\begin{aligned} \phi_{SSB} &= \cos(100t) \cos(1000t) \mp \sin(100t) \sin(1000t) \\ \text{by } \cos(x) \cos(y) &= \frac{1}{2} [\cos(x-y) + \cos(x+y)] \text{, } \sin(x) \sin(y) = \frac{1}{2} [\cos(x-y) - \cos(x+y)] \\ &= \frac{1}{2} (\cos(900t) + \cos(1100t)) \mp \frac{1}{2} (\cos(900t) - \cos(1100t)) \end{aligned}$$

$$= \frac{1}{2} [\cos(900t) \mp \cos(900t) + \cos(1100t) \pm \cos(1100t)]$$

$$\phi_{USB} = \frac{1}{2} [0 + 2 \cos(1100t)] = \cos(1100t)$$

$$\phi_{LSB} = \frac{1}{2} [2 \cos(900t) + 0] = \cos(900t)$$


---

$$4.5-2 \quad (b) \quad m(t) = \cos(100t) + 2 \cos(300t)$$

$$m_h(t) = \sin(100t) + 2 \sin(300t)$$

$$\begin{aligned} \phi_{SSB} &= [\cos(100t)\cos(1000t) \mp \sin(100t)\sin(1000t)] \\ &\quad + 2 [\cos(300t)\cos(1000t) \mp \sin(300t)\sin(1000t)] \\ &= \frac{1}{2} \left[ \overset{1000-100}{\downarrow} [\cos(900t) + \overset{1000+100}{\downarrow} \cos(1100t)] \mp [\cos(900t) - \cos(1100t)] \right] \\ &\quad + \frac{1}{2} \left[ \overset{1000-300}{\downarrow} [\cos(700t) + \cos(1300t)] \mp [\cos(700t) - \cos(1300t)] \right] \end{aligned}$$

$$\phi_{USB} = \frac{1}{2} [0 + 2 \cos(1100t)] + [0 + 2 \cos(1300t)]$$

$$= \cos(1100t) + 2 \cos(1300t)$$

$$\phi_{LSB} = \frac{1}{2} [2 \cos(900t) + 0] + [2 \cos(700t) + 0]$$

$$= \cos(900t) + 2 \cos(700t)$$

4.5-2

$$(C) \quad m(t) = \cos(100t) \cos(500t)$$

$$= \frac{1}{2} [\cos(400t) + \cos(600t)]$$

$$m_H(t) = \frac{1}{2} [\sin(400t) + \sin(600t)]$$

$$\phi_{SSB} = \frac{1}{2} [\cos(400t) + \cos(600t)] \cos(1000t)$$

$$\mp \frac{1}{2} [\sin(400t) + \sin(600t)] \sin(1000t)$$

$$= \frac{1}{2} \left[ \cos(400t) \cos(1000t) + \cos(600t) \cos(1000t) \right]$$

$$\mp \left[ \sin(400t) \sin(1000t) + \sin(600t) \sin(1000t) \right]$$

$$= \frac{1}{2} \left[ \frac{1}{2} [\cos(600t) + \cos(1400t)] + \frac{1}{2} [\cos(400t) + \cos(1600t)] \right]$$

$$\mp \frac{1}{2} [\cos(600t) - \cos(1400t)] + \frac{1}{2} [\cos(400t) - \cos(1600t)]$$

$$= \frac{1}{4} [\cos(600t) \mp \cos(600t) + \cos(1400t) \pm \cos(1400t)]$$

$$+ \cos(400t) \mp \cos(400t) + \cos(1600t) \pm \cos(1600t)]$$

$$\phi_{USB} = \frac{1}{4} [0 + 2\cos(1400t) + 0 + 2\cos(1600t)]$$

$$= \frac{1}{2} [\cos(1400t) + \cos(1600t)]$$

$$\phi_{LSB} = \frac{1}{4} [2\cos(600t) + 0 + 2\cos(400t) + 0]$$

$$= \frac{1}{2} [\cos(600t) + \cos(400t)]$$

4.5-3 Find  $\phi_{LSB}$ ,  $\phi_{USB}$  for  $m(t) = B \text{sinc}(2\pi Bt)$

$$B = 1000$$

$$\omega_c = 10000\pi$$

- Steps: (a) sketch  $m(t)$  <sup>spectrum</sup> and DSB-SC signal  $2m(t)\cos\omega_c t$   
 (b) Find LSB by suppressing USB component  
 (c) Take inverse Fourier transform to find  $\phi_{LSB}$

(a) Using Transform pair #18

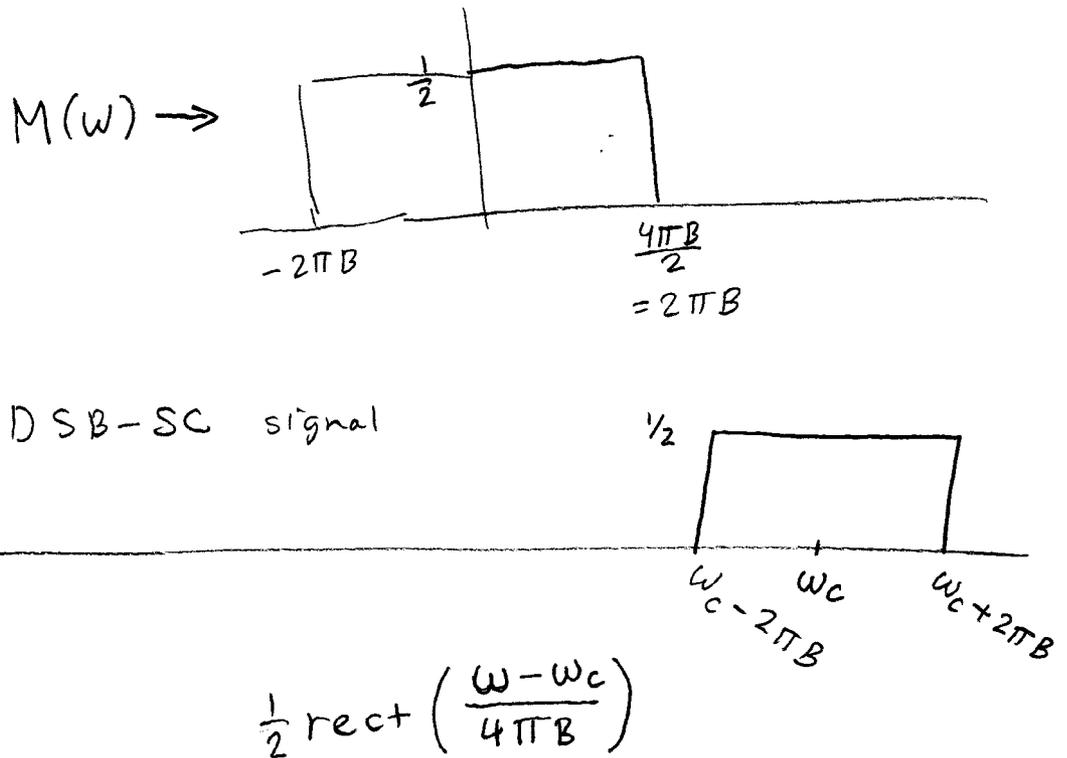
$$\frac{W}{\pi} \text{sinc}(Wt) \Leftrightarrow \text{rect}\left(\frac{W}{2W}\right)$$

$$\text{Let } W = 2\pi B$$

$$\text{then } \frac{W}{\pi} = 2B$$

$$\text{so } 2B \text{sinc}(2\pi Bt) \Leftrightarrow \text{rect}\left(\frac{W}{2 \cdot 2\pi B}\right)$$

$$B \text{sinc}(2\pi Bt) \Leftrightarrow \frac{1}{2} \text{rect}\left(\frac{W}{4\pi B}\right)$$



4.5-3

(b) Find LSB by suppressing USB component



$$= \frac{1}{2} \text{rect} \left( \frac{\omega - (\omega_c - \pi B)}{2\pi B} \right)$$

Use Frequency shifting:

$$\frac{\pi B}{\pi} \text{sinc}(\pi B t) \iff \text{rect} \left( \frac{\omega}{2\pi B} \right)$$

$\omega = \pi B$

$$\text{Envelope} = B \text{sinc}(\pi B t)$$

$$\frac{1}{2} B \text{sinc}(\pi B t) \iff \frac{1}{2} \text{rect} \left( \frac{\omega}{2\pi B} \right)$$

Now frequency shift --

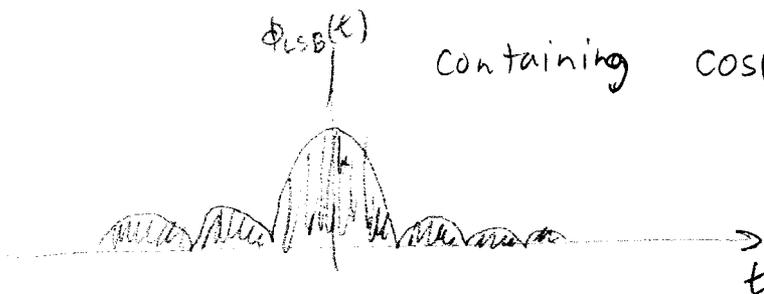
$$g(t) e^{j\omega_0 t} \iff G(\omega - \omega_0)$$

$\omega_0 = \omega_c - \pi B$

$$\phi_{\text{LSB}}(t) = \frac{1}{2} B \text{sinc}(\pi B t) e^{j(\omega_c - \pi B)t}$$

$\frac{1}{2} B \text{sinc}(\pi B t)$  envelope

containing  $\cos((\omega_c - \pi B)t)$



4.5-5 An LSB signal is demodulated with an error.

Transmit "carrier" was  $2 \cos \omega_c t$

but receive BFO is  $2 \cos[(\omega_c + \Delta\omega)t + \delta]$

Show that:

(a) when  $\delta = 0$ , output  $y(t)$  has all components shifted by  $\Delta\omega$ .

See class notes -- 4B-3

Here's time domain approach.

Suppose  $m(t) = \cos(\omega_m t)$  --

The LSB signal is  $\cos((\omega_c - \omega_m)t)$

To demodulate --- multiply by  $2 \cos(\omega_c t)$

but actually use  $2 \cos((\omega_c + \Delta\omega)t)$

$$\begin{aligned} y(t) &= \cos((\omega_c - \omega_m)t) 2 \cos((\omega_c + \Delta\omega)t) \\ &= 2 \cos((\omega_c - \omega_m)t) \cos((\omega_c + \Delta\omega)t) \\ &= 2 \left( \frac{1}{2} (\cos((\omega_c - \omega_m - \omega_c - \Delta\omega)t) + \cos((\omega_c - \omega_m + \omega_c + \Delta\omega)t)) \right) \\ &= \underbrace{\cos((- \omega_m - \Delta\omega)t)}_{\text{Keep this}} + \underbrace{\cos((2\omega_c - \omega_m + \Delta\omega)t)}_{\text{Filter out.}} \\ &= \cos((\omega_m + \Delta\omega)t) \\ &\quad \uparrow \\ &\quad \text{frequency shift} \end{aligned}$$

4.5-5 - b. when  $\Delta\omega = 0$ , output is  $m(t)$  phase shifted by  $\delta$

To demodulate - multiply by  $2 \cos(\omega_c t)$   
but actually use  $2 \cos(\omega_c t + \delta)$

$$\begin{aligned}y(t) &= \cos((\omega_c - \omega_m)t) 2 \cos(\omega_c t + \delta) \\&= 2 \cos((\omega_c - \omega_m)t) \cos(\omega_c t + \delta) \\&= 2 \left( \frac{1}{2} (\cos(\omega_c t - \omega_m t - \omega_c t - \delta) + \cos(\omega_c t - \omega_m t + \omega_c t + \delta)) \right) \\&= \underbrace{\cos(-\omega_m t - \delta)}_{\text{Keep this}} + \underbrace{\cos(2\omega_c t - \omega_m t + \delta)}_{\text{Filter out}} \\&= \cos(\omega_m t + \delta) \\&\quad \uparrow \\&\quad \text{phase shift}\end{aligned}$$

4.5-6

An USB signal is generated by phase shift method  
if input is  $m_h(t)$  instead of  $m(t)$   
What is output?

---

Since  $m_h(t)$  is  $m(t)$  with  $90^\circ$  delay

Apply Hilbert transform again yields  $-m(t)$   
( $180^\circ$  delay)

4.17a says:  $\Phi_{USB} = m(t) \cos(\omega_c t) - m_h(t) \sin(\omega_c t)$

Substitute --  $\boxed{\text{output} = m_h(t) \cos(\omega_c t) + m(t) \sin(\omega_c t)}$

it is still USB, with same bandwidth.

To get back  $m(t)$  phase shift it back --

or apply the opposite phase shift to  $\omega_c$  --

$$-m_h(t) \sin(\omega_c t) + m(t) \cos(\omega_c t)$$