

4B-1

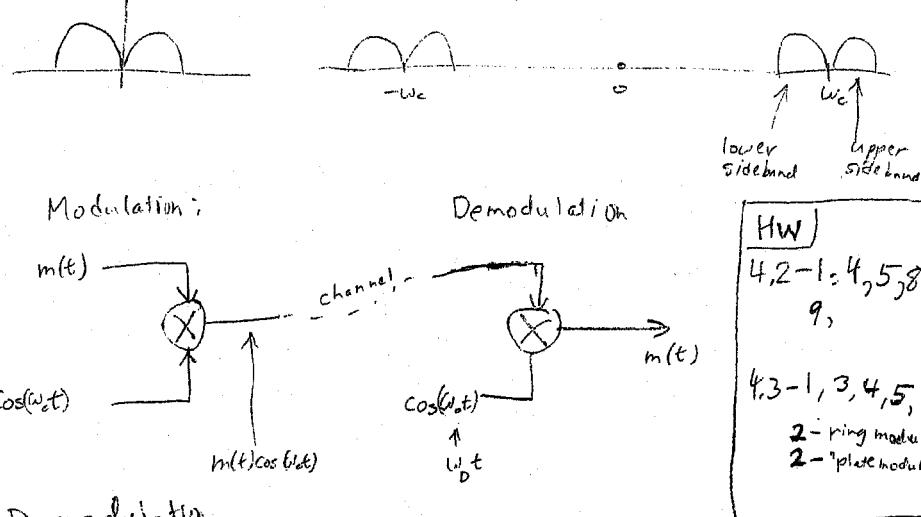
Double Sideband - Suppressed-carrier

Math! recall frequency shifting:

$$m(t) \Leftrightarrow M(w)$$

$$m(t) \cos(\omega_c t) \Leftrightarrow \frac{1}{2} [M(w+\omega_c) + M(w-\omega_c)]$$

$$M(w) =$$



Demodulation

$$(m(t)\cos(\omega_c t))\cos(\omega_d t) \Leftrightarrow \frac{1}{2} \left[\frac{1}{2} [M(w+\omega_c+\omega_d) + M(w-\omega_c+\omega_d)] \right. \\ \left. + \frac{1}{2} [M(w+\omega_c-\omega_d) + M(w-\omega_c-\omega_d)] \right]$$

$$\text{Let } \omega_d = \omega_b;$$

$$= \frac{1}{2} \left[\frac{1}{2} [M(w+2\omega_c) + M(w)] \right. \\ \left. + \frac{1}{2} [M(w) + M(w-2\omega_c)] \right]$$

$$= \frac{1}{4} [M(w+2\omega_c) + M(w-2\omega_c) + 2M(w)]$$

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Note that:

- Neither input appears at the output of the detector.
- The baseband ($M(w)$) is the desired output.
- The product of the carriers (2ω) also appears at the output.
This is easy to filter out.
Simple low pass filter.

- The "local oscillator" is critical.
Any error will produce distortion of the recovered signal.

→ Envelope detection doesn't work.

→ This new type of detection is called ...

"synchronous detection" — a local carrier is synchronised with the transmitting carrier

"product detector" — It is built around a multiplier — hence "product".

What happens if the receiver's carrier (local oscillator, "BFO" = "beat frequency oscillator") is a little off --

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① Try $w_0 = w_c + 1$ (1 Hz error--).

$$\begin{aligned} & \frac{1}{2} \left[\frac{1}{2} [M(w+w_c+w_c+1) + M(w-w_c+w_c+1)] \right. \\ & \quad \left. + \frac{1}{2} [M(w+w_c-w_c-1) + M(w-w_c-w_c-1)] \right] \\ & = \frac{1}{4} \left[\underbrace{M(w+2w_c+1)}_{2w - \text{filtered out}} + \underbrace{M(w-2w_c-1)}_{\text{bad Approximation}} + M(w+1) + M(w-1) \right] \end{aligned}$$

↓
bad Approximation
of modulating signal.

$$K M(w) \neq M(w+1) + M(w-1)$$

↑
arbitrary
constant

If $M(w)$ was 1000 Hz

We receive 999 Hz
and 1001 Hz instead.

Sounds like 1000 Hz with 2 Hz modulation.

② What if receiver has $\sin(w_0 t)$ instead of $\cos(w_0 t)$?

Fourier transforms:

$$\cos(w_0 t) \Leftrightarrow \pi [\delta(w+w_0) + \delta(w-w_0)]$$

$$\sin(w_0 t) \Leftrightarrow (\text{j})\pi [\delta(w+w_0) - \delta(w-w_0)]$$

$$g(t) \cos(w_0 t) \Leftrightarrow \frac{1}{2} [G(w+w_0) + G(w-w_0)]$$

$$g(t) \cos(w_0 t + \theta_0) \Leftrightarrow \frac{1}{2} [G(w+w_0)e^{-j\theta_0} + G(w-w_0)e^{+j\theta_0}]$$

Let $\theta_0 = -\frac{\pi}{2}$, this means $\cos(w_0 t + \frac{\pi}{2}) = \sin(w_0 t)$

$$g(t) \sin(w_0 t) \Leftrightarrow \frac{1}{2} [G(w+w_0)e^{j\frac{\pi}{2}} + G(w-w_0)e^{-j\frac{\pi}{2}}]$$

It's all phase shifted.
Now, try modulating with cos, demod with sin--

$$\begin{aligned} & \frac{1}{2} \left[\frac{1}{2} [M(w+2w_c) + M(w)] e^{j\frac{\pi}{2}} + \frac{1}{2} [M(w) + M(w-2w_c)] e^{-j\frac{\pi}{2}} \right] \\ & = \frac{1}{4} \left[M(w+2w_c)e^{j\frac{\pi}{2}} + M(w-2w_c)e^{-j\frac{\pi}{2}} + M(w)e^{j\frac{\pi}{2}} + M(w)e^{-j\frac{\pi}{2}} \right] \end{aligned}$$

↓ ↓ ↓ ↓
+90° -90°

Idea: "Quadrature AM" (4.4)

Transmit 2 signals on the same frequency --
one with $\cos(w_0 t)$, other with $\sin(w_0 t)$.
"I" (in phase) "Q" (quadrature)
Detect with same phasor... Recover 2 signals.

Used for: color in TV.

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