

Test : Background math, etc.

3A-0

Energy & power

Thursday

Fourier series, Fourier transform

Properties (shifting, linearity, duality, etc.)

Finding  $G(\omega)$  from  $g(t)$

$g(t)$  from  $G(\omega)$

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(1D) 2.1 - 1, 2, 5      Homework summary

3 - 1, 2, 3, 4

5 - 2, 5

6 - 1

8 - 2

(2A) 2.8 - 4

9 - 1

(2B) 3.1 - 4, 5, 6

3 - 1, 2, 3, 6, 7

(3A) 3.7 - 5

Defer correlation.

3A-1

Energy, energy spectral density,  
"essential bandwidth", etc.

Recall -- Parseval's theorem -.

$$E_g = \sum C_n^2 E_n \quad \leftarrow \text{Energy of a signal is the sum of the energy of the components.}$$

(p. 43)

Definition of Energy.

Energy is:

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

For complex signals --

$$= \int_{-\infty}^{\infty} g(t) g^*(t) dt$$

because  $|g(t)|^2 = g(t)g^*(t)$

If we have  $G(\omega)$ , we can compute energy this way:

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega$$

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HW: 3.7 - 5

Derivation:

3A-2

$$\text{Start here: } E_g = \int_{-\infty}^{\infty} g(t) g^*(t) dt$$

$$\text{sub for } g^*(t): E_g = \int_{-\infty}^{\infty} g(t) \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} G^*(\omega) e^{-j\omega t} d\omega \right] dt$$

Interchange order of integration:

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} G^*(\omega) \left[ \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt \right] d\omega$$

Sub for  $\int g(t) e^{-j\omega t} dt$

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) G^*(\omega) d\omega$$

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega$$

3A-3

Energy Spectral Density (ESD)

(or -- where is that energy?)

Take a filter, sweep it ...

ps?  $\rightarrow \Psi_g(\omega) = |G(\omega)|^2$

(or -- according to some --)

$$\Psi_g(\omega) = 2 |G(\omega)|^2$$

"Essential bandwidth"

or -- how much space does that signal really take up?

If a real signal has infinite bandwidth,  
what band contains most of the energy?

"most" must be defined.

Example Try book example (p.118) but for 99%. 3A-4

The signal:

$$g(t) = e^{-at} u(t)$$

The Fourier transform:

$$G(w) = \frac{1}{a + jw} \quad \text{from table p.85}$$

The ESD:

$$\psi(w) = |G(w)|^2 = \frac{1}{a^2 + w^2}$$

The energy in the signal:

$$E_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(w)|^2 dw$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{a^2 + w^2} dw$$

$$= \frac{1}{2\pi a} \left[ \tan^{-1} \frac{w}{a} \right]_{-\infty}^{\infty} \quad \begin{matrix} \text{from integral table} \\ \text{p. 774} \end{matrix}$$

$$= \frac{1}{2\pi a} \left( \tan^{-1} \frac{\infty}{a} - \tan^{-1} \frac{-\infty}{a} \right)$$

$$= \frac{1}{2\pi a} \left( \frac{\pi}{2} - \left( -\frac{\pi}{2} \right) \right)$$

$$E_g = \frac{1}{2a}$$

We want to find limits on the integral such that  $E_{(99)} = 0.99 E_g$  3A-5

$$\frac{0.99}{2a} = \frac{1}{2\pi} \int_{-W}^W \frac{1}{a^2 + w^2} dw$$

$$\frac{0.99}{2a} = \frac{1}{2\pi a} \left[ \tan^{-1} \frac{w}{a} \right]_{-W}^W$$

$$\frac{0.99}{2a} = \frac{1}{2\pi a} \left( \tan^{-1} \frac{W}{a} - \tan^{-1} \frac{-W}{a} \right)$$

$$= \frac{1}{2\pi a} \left( 2 \tan^{-1} \frac{W}{a} \right)$$

$$\frac{0.99}{2a} = \frac{1}{\pi a} \left( \tan^{-1} \frac{W}{a} \right)$$

$$\frac{(0.99)(\pi a)}{2a} = \tan^{-1} \frac{W}{a}$$

$$\frac{0.99\pi}{2} = \tan^{-1} \frac{W}{a}$$

$$\tan \frac{0.99\pi}{2} = \frac{W}{a}$$

$$a \tan \frac{0.99\pi}{2} = W$$

be sure to use radians

$$W = 63.66 a \text{ rad/sec}$$

Contrast to 12.706 a for 95%