

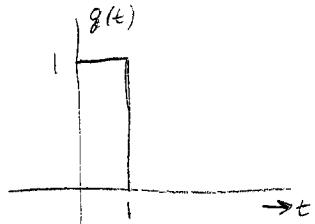
More Fourier Transform

2B-1

- Finish derivation (2A-9, 2A-10)

Fourier series - look at text p 44, 50, 52 - critical freq.

Example: Find the Fourier transform of:



$$g(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$= \int_0^1 e^{-j\omega t} dt$$

$$= -\frac{1}{j\omega} e^{-j\omega t} \Big|_0^1$$

$$= \left(-\frac{1}{j\omega} e^{-j\omega} \right) - \left(-\frac{1}{j\omega} e^0 \right)$$

$$= -\frac{1}{j\omega} e^{-j\omega} + \frac{1}{j\omega}$$

$$= \frac{1}{j\omega} \left(1 - e^{-j\omega} \right)$$

octave pgm:

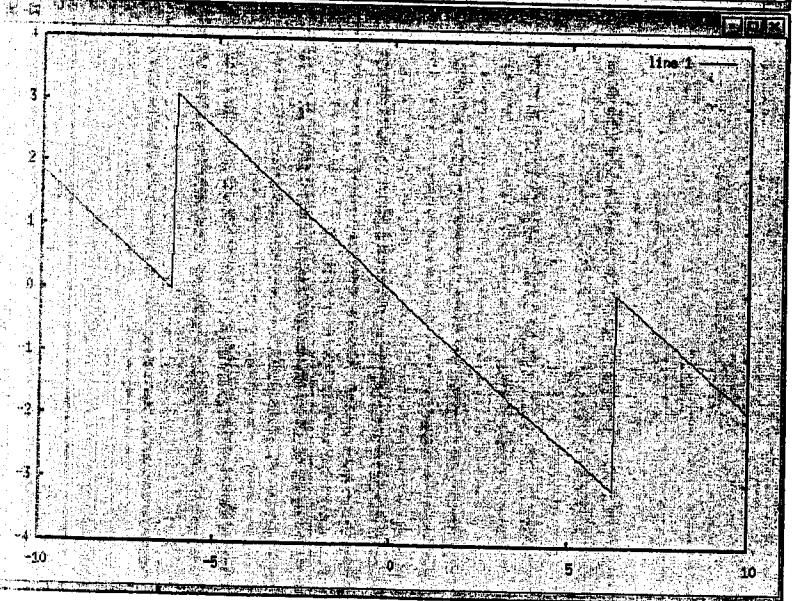
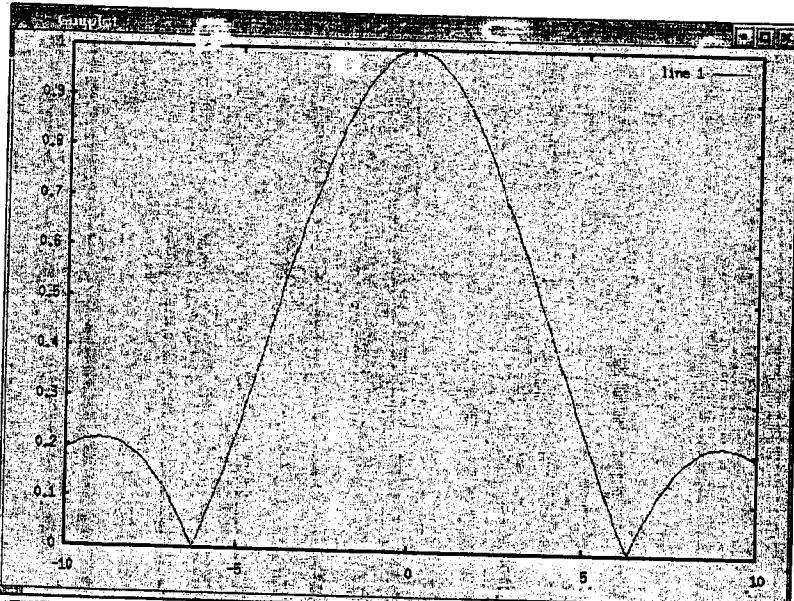
$w = -100:100;$

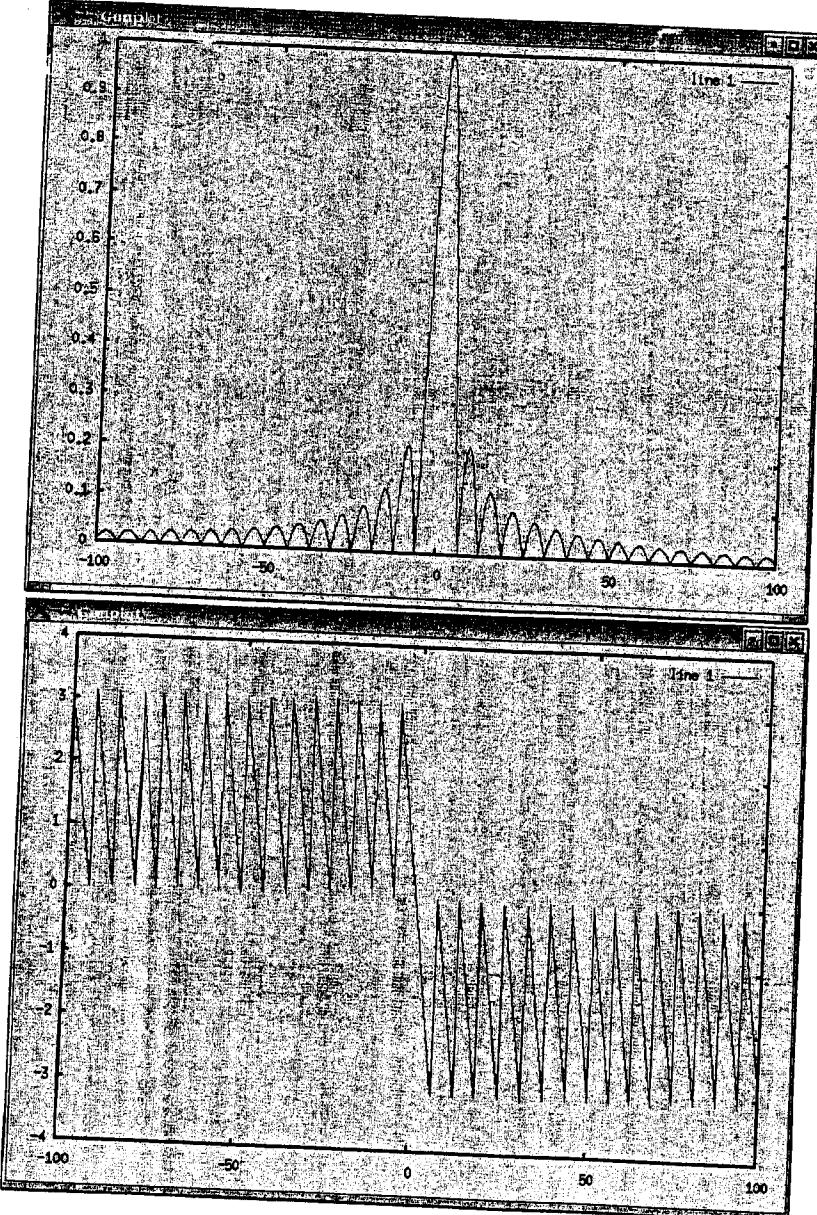
$g = 1 ./ (1 - exp(-j*w))$

$\star(1 - exp(-j*w))$

$\text{plot}(w, abs(g))$

$\text{plot}(w, \text{arg}(g))$



Existence:

Fourier transform does not exist for all signals.

Example: growing exponential.

(See Example 3.1 in text.)

$$\text{It exists when } \int_{-\infty}^{\infty} |g(t)| dt < \infty$$

(the integral is finite)

Linearity:

It is a linear operation.

Suppose we have Fourier transform pairs:

$$g_1(t) \Leftrightarrow G_1(\omega)$$

$$\text{and } g_2(t) \Leftrightarrow G_2(\omega)$$

Has Fourier transform

Then:

$$a_1 g_1(t) + a_2 g_2(t) \Leftrightarrow a_1 G_1(\omega) + a_2 G_2(\omega)$$

↓

The sum of
four signals

The sum of the
Fourier transforms

Significance -- If we have a complex signal,
we can break it into the sum of parts,
take FT of each part. The FT of
the total is the sum of the parts.

Physical significance of the F.T.

2B-5

It represents a signal in terms of sinusoids or exponentials.

It indicates relative amplitude and phase of the sinusoids required to build that signal.

$G(\omega)$ is the spectral density per unit bandwidth (in Hz)

it is also called the spectrum

If a signal is ...

periodic

A periodic

discrete

continuous

The spectrum is ...

discrete

continuous

periodic

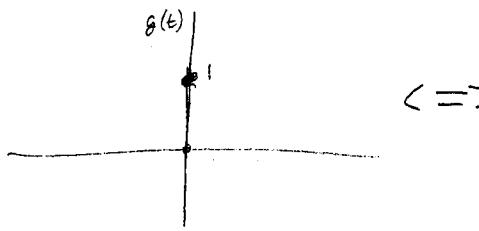
A periodic

Some typical transforms ..

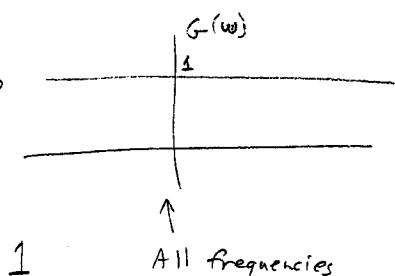
See table p. 85

2B-6

Impulse



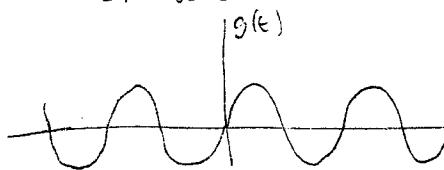
\Leftrightarrow



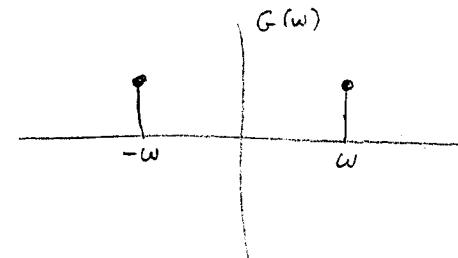
$s(t) \Leftrightarrow 1$

All frequencies

Sinusoid

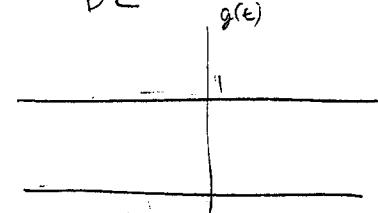


$g(t)$

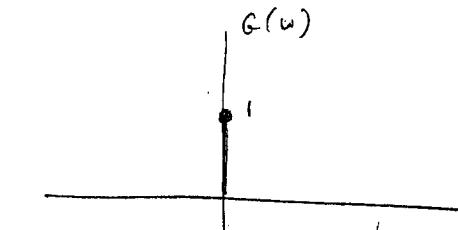


$G(w)$

DC

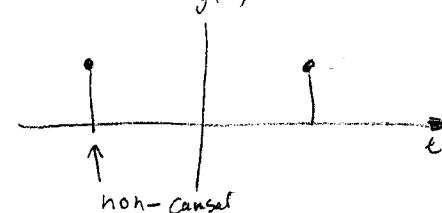


$g(t)$

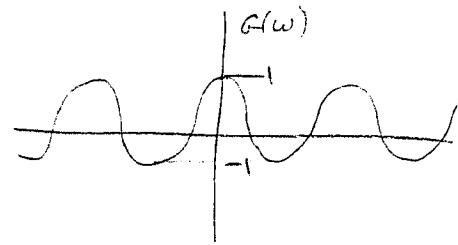


$G(w)$

$g(t)$



non-causal



$G(w)$

Properties

2B-7

Duality: The direct and inverse transforms are similar.

They differ only by a sign and a scale factor.

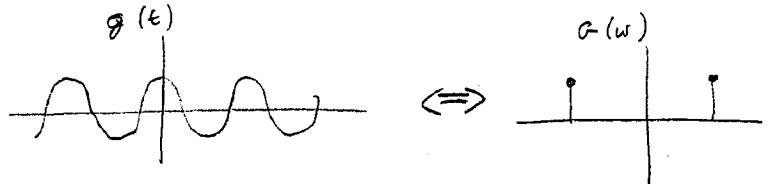
For any relationship between $g(t)$ and $G(w)$

There exists a similar relationship interchanging them.

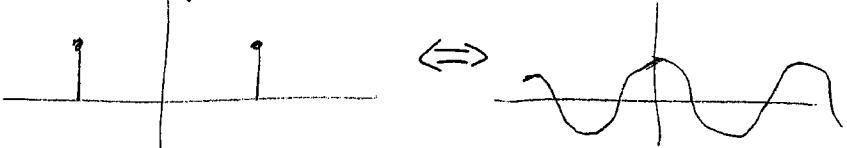
Symmetry

If: $g(t) \Leftrightarrow G(w)$

then: $G(t) \Leftrightarrow 2\pi g(-w)$



implies that:



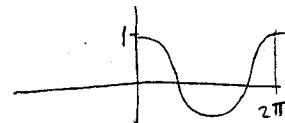
Scaling

if: $g(t) \Leftrightarrow G(w)$

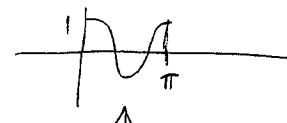
then: $g(at) \Leftrightarrow \frac{1}{|a|} G\left(\frac{w}{a}\right)$

2B-8

Example: $g(t) = \cos(t) \Leftrightarrow G(w) = \begin{cases} \pi & \pm 1 \\ 0 & \text{elsewhere} \end{cases}$



then $(a=2)$
 $g(2t) = \cos(2t) \Leftrightarrow \frac{1}{2} G\left(\frac{w}{2}\right) = \begin{cases} (\frac{1}{2})(2\pi) & \pm 2 \\ 0 & \text{elsewhere} \end{cases}$



Signal is compressed in t
↑
Spectrum is expanded.

$$\pi [\delta(w-1) + \delta(w+1)]$$

Time shiftingif $g(t) \Leftrightarrow G(w)$

2B-9

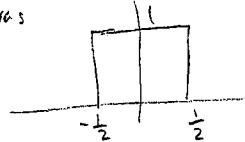
then $g(t-t_0) \Leftrightarrow G(w)e^{-jw_0t_0}$

Time shift, in time domain

results in phase shift
in frequency domain

SEE the example in today's notes - p.1

What if it was

Amplitude would be the same,
but phase would be 0.

Linear phase shift = time delay.

Frequency shiftingif $g(t) \Leftrightarrow G(w)$

then $g(t)e^{jw_0t} \Leftrightarrow G(w-w_0)$
results in "modulation" Frequency shift

also -- $g(t)e^{-jw_0t} \Leftrightarrow G(w+w_0)$

2B-9

More frequency shifting --

2B-10

Recall — e^{jw_0t} is a sinusoid \leftrightarrow

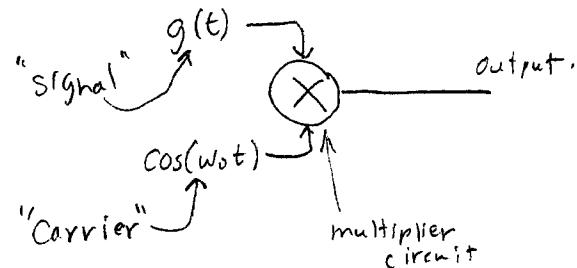
$\cos(w_0t) = \frac{1}{2}(e^{jw_0t} + e^{-jw_0t})$

so ...

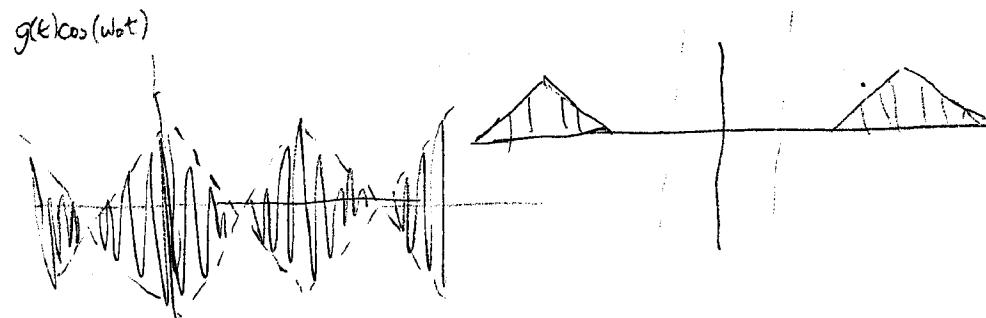
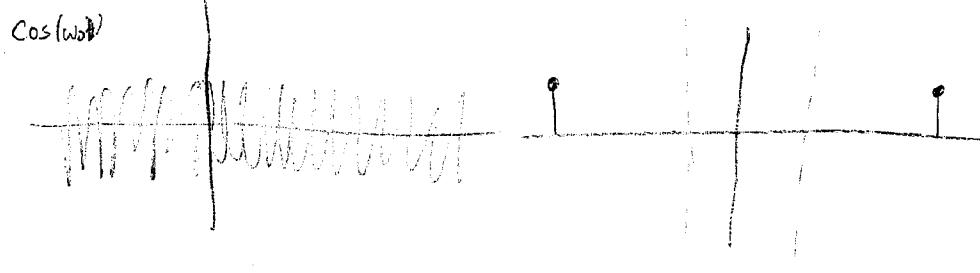
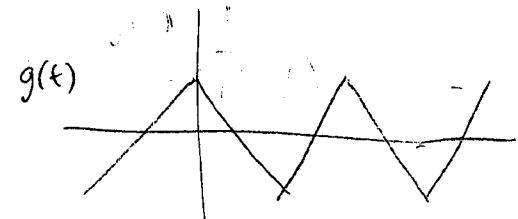
$g(t) \cos(w_0t) = \frac{1}{2}(g(t)e^{jw_0t} + g(t)e^{-jw_0t})$



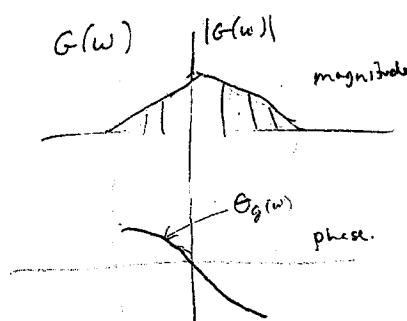
$g(t) \cos(w_0t) \Leftrightarrow \frac{1}{2}(G(w-w_0) + G(w+w_0))$

Frequency shifting is multiplication by
a sinusoid.

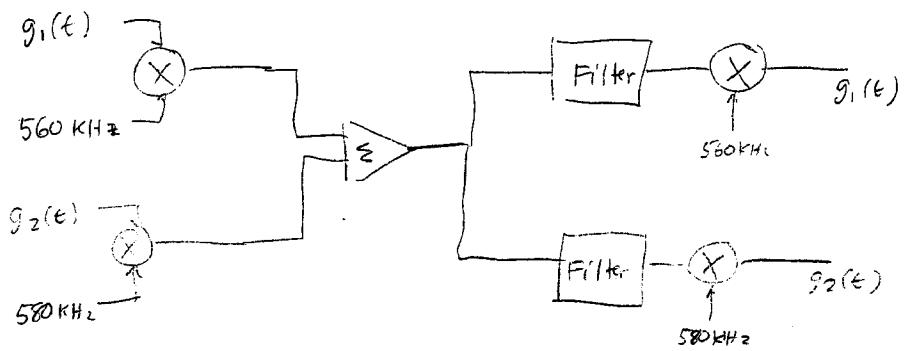
The multiplier does this:



2B-11



112 can frequency shift a signal,
so several band-limited signals
can share a channel -
"Frequency division multiplexing"



Band pass signals --
This works when $g(t)$ is band-limited.

Otherwise the sidebands can overlap --
aliasing.