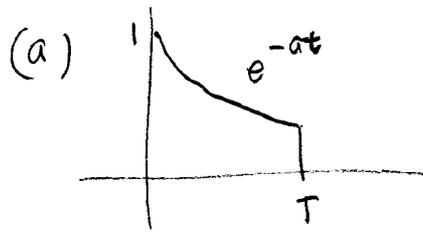


3.1-4 Find Fourier transform from definition.



$$G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$= \int_0^T e^{-at} e^{-j\omega t} dt$$

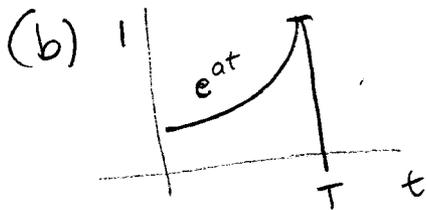
$$= \int_0^T e^{-(a-j\omega)t} dt$$

$$= \frac{1}{-a-j\omega} \left(e^{-(a-j\omega)t} \right) \Big|_0^T$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

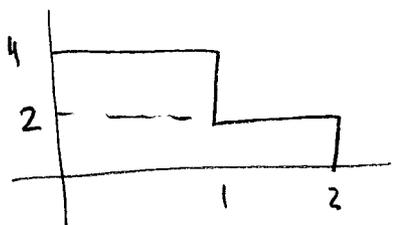
$$= \frac{1}{-a-j\omega} \left(e^{-(a-j\omega)T} - e^0 \right)$$

$$G(\omega) = \frac{1}{a+j\omega} (1 - e^{-(a-j\omega)T})$$



3.1-5 Find Fourier transforms -

(a)



$$g(t) = \begin{cases} 4 & 0 < t < 1 \\ 2 & 1 < t < 2 \\ 0 & t < 0, t > 2 \end{cases}$$

$$G(\omega) = \int_0^1 4e^{-j\omega t} dt + \int_1^2 2e^{-j\omega t} dt$$

$$= 4 \left. \frac{1}{-j\omega} e^{-j\omega t} \right|_0^1 + 2 \left. \frac{1}{-j\omega} e^{-j\omega t} \right|_1^2$$

$$= \frac{4}{-j\omega} (e^{-j\omega} - e^0) + \frac{2}{-j\omega} (e^{-j2\omega} - e^{-j\omega})$$

$$= \frac{-2}{j\omega} (2e^{-j\omega} - 2 + e^{-j2\omega} - e^{j\omega})$$

$$= \frac{-2}{j\omega} (e^{-j2\omega} + e^{-j\omega} - 2)$$

3.3-1

$$(b) = \delta(t+T) + \delta(t-T)$$

$$\stackrel{?}{\Leftrightarrow} 2 \cos T\omega$$

Symmetry says:

$$\text{if } g(t) \Leftrightarrow G(\omega)$$

$$\text{then } G(t) \Leftrightarrow 2\pi g(\omega)$$

$$\#9: \cos \omega_0 t \Leftrightarrow \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

by symmetry:

$$\pi [\delta(t - t_0) + \delta(t + t_0)] \Leftrightarrow 2\pi \cos t_0 \omega$$

divide by π ; sub T for t_0

$$[\delta(t - T) + \delta(t + T)] \Leftrightarrow 2 \cos T\omega$$

(c) use #10

$$(a) .5 \left[\delta(t) + \left(\frac{j}{\pi t} \right) \right] \Leftrightarrow ?$$

by symmetry, #11

$$\pi \delta(t) + \frac{j}{t} \Leftrightarrow 2\pi u(\omega)$$

divide by π

$$\delta(t) - \frac{j}{\pi t} \Leftrightarrow 2 u(\omega)$$

mult by .5

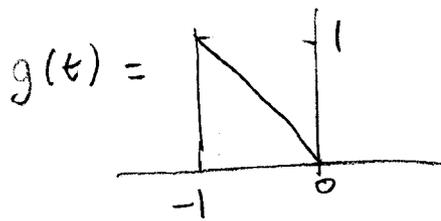
$$.5 \left[\delta(t) - \frac{j}{\pi t} \right] \Leftrightarrow u(\omega)$$

$$g(-t) \Leftrightarrow G(-\omega)$$

$$.5 \left[\delta(-t) + \frac{j}{\pi t} \right] \Leftrightarrow u(\omega)$$

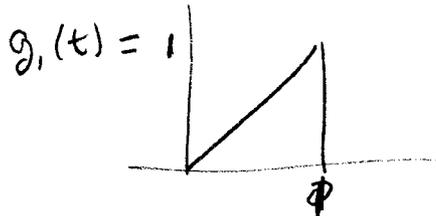
$$.5 \left[\delta(t) + \frac{j}{\pi t} \right] = u(\omega)$$

3-3-2



$$\Leftrightarrow G(\omega) = \frac{1}{\omega^2} (e^{j\omega} - j\omega e^{j\omega} - 1)$$

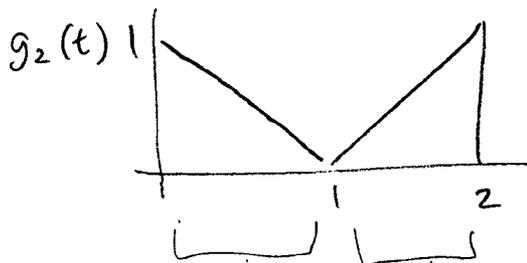
by properties ---



$$g_1(t) = g(-t)$$

$$G_1(\omega) = G(-\omega)$$

$$= \frac{1}{\omega^2} (e^{-j\omega} + j\omega e^{-j\omega} - 1)$$



$$g_2(t) = g(t-1) + g_1(t-1)$$

$$G_2(\omega) = \left[\frac{1}{\omega^2} (e^{j\omega} - j\omega e^{j\omega} - 1) + \frac{1}{\omega^2} (e^{-j\omega} + j\omega e^{-j\omega} - 1) \right] e^{-j\omega}$$

3.3-6

$$\text{carrier} = \cos(10t)$$

$$(a) \quad g(t) = \begin{cases} 1 - \frac{t}{\pi} & 0 \leq t < \pi \\ 1 + \frac{t}{\pi} & -\pi < t < 0 \\ 0 & \text{elsewhere} \end{cases} = \Delta\left(\frac{t}{\pi}\right)$$

$$y(t) = \Delta\left(\frac{t}{\pi}\right) \cos(10t)$$

by frequency shifting --

$$g(t) \cos(\omega_0 t) \Leftrightarrow \frac{1}{2} (G(\omega - \omega_0) + G(\omega + \omega_0))$$

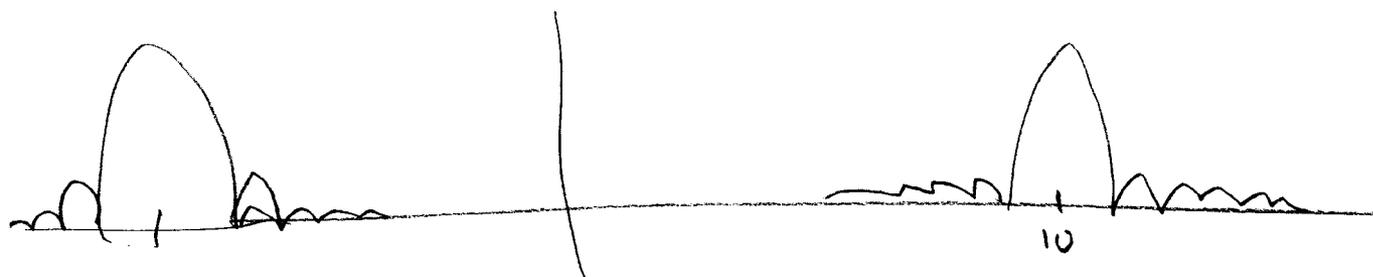
$$\omega_0 = 10$$

$$\text{F.T. #19: } \Delta\left(\frac{t}{\tau}\right) \Leftrightarrow \frac{\tau}{2} \text{sinc}^2\left(\frac{\omega\tau}{4}\right) \quad \tau = \pi$$

$$G(\omega) = \frac{\pi}{2} \text{sinc}^2\left(\frac{\omega\pi}{4}\right)$$



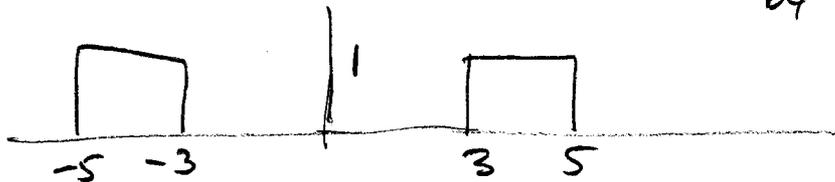
$$Y(\omega) = \frac{1}{2} \left[\frac{\pi}{2} \text{sinc}^2\left(\frac{(\omega-10)\pi}{4}\right) + \frac{\pi}{2} \text{sinc}^2\left(\frac{(\omega+10)\pi}{4}\right) \right]$$



3,3-7

Find inverse F.T. of ...

by frequency shifting.

 $G(\omega)$ 

$$\frac{W}{\pi} \text{sinc}(Wt) \Leftrightarrow \text{rect}\left(\frac{\omega}{2W}\right)$$

$W=1$

$$= \text{rect}\left(\frac{\omega}{2}\right) \Leftrightarrow \frac{1}{\pi} \text{sinc}(t)$$

$$G(\omega) = \text{rect}\left(\frac{\omega-4}{2}\right) + \text{rect}\left(\frac{\omega+4}{2}\right)$$

by frequency shifting: $g(t) \cos(\omega_0 t) \Leftrightarrow \frac{1}{2} [G(\omega - \omega_0) + G(\omega + \omega_0)]$

$$g(t) = \frac{1}{\pi} \text{sinc}(t) \cos 4t$$