Basics of signals

(more basic than text, review?)

Energy, power, etc.

Types of signals

etc.

Read –
Ch. 1 Sec 1-8
skip sec 9-11
(digital,
do it then)

Ch. 2 -
sec 1 - basics
4
5
2
3
7
8
Any order

Classification of signals

continuous time vs. discrete time (samples)

Analog (continuous magnitude) vs. digital (discrete magnitude)

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periodic (repeats)

$g(t) = g(t+T_0)$ for all t

vs. aperiodic (never repeats)

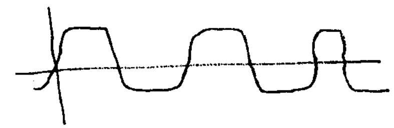
Deterministic vs. **Random**

Deterministic:
Known completely
Can be expressed completely in mathematical or graphical form

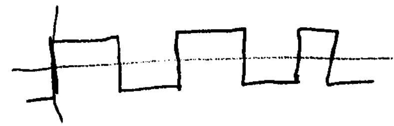
Random:
Known only in probabalistic sense
Most noise is random (but not all)
All useful message signals are random
(If it is deterministic, why bother sending it?)

Physically realizable waveforms (analog)

Realizable



Not realizable



	Realizable	Non realizable
①	Significant non-zero values over a finite time interval, zero (or don't care) outside.	Extends to $\pm\infty$ in time.
②	Spectrum has significant values over finite interval. Zero (or don't care) outside	Extends to $\pm\infty$ in frequency
③	Continuous in time	Discrete in time.
④	Finite peak value	Infinite peak value
⑤	Only real values	Complex values.

↑ interesting mathematically, but don't exist physically. Use approximations.

We will make believe that all waveforms are periodic.

Operations --- (basics, terminology, etc, ..)

Time average .. = DC value.

$$\underbrace{\langle[\bullet]\rangle}_{\text{defining this operator}} = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} [\bullet]\, dt$$

$[\bullet]$ means "some expression"

$\langle \sin(x) \rangle$ means "time average of $\sin(x)$"

is calculated as : $\lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sin(x)\, dt$

If waveform is periodic
($w(t) = w(t+T_0)$

$w(t)$ means "some waveform"

we don't need to take the limit --
just do it over one period.

$$\langle[\bullet]\rangle = \frac{1}{T_0}\int_{a-\frac{T_0}{2}}^{a+\frac{T_0}{2}} [\bullet]\, dt$$

$$= \frac{1}{T_0}\int_{a}^{a+T_0} [\bullet]\, dt$$

For Practical waveform, we only care from t_1 to t_2

$$\langle[\bullet]\rangle = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} [\bullet]\, dt$$

(make believe it is periodic).

Energy + power

Instantaneous power : $p(t) = v(t)\, i(t)$

Average power : $P = \langle p(t)\rangle = \langle v(t)\, i(t)\rangle$

$$= \frac{1}{T_0}\int_{a}^{a+T_0} v(t)\, i(t)\, dt$$

Watts

"Normalized" power : —

measure only voltage,
assume a $1\,\Omega$ load, so $I = V$.

(really cheating, but when you need only a ratio, the final result is correct)

$$P = \langle w^2(t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} w^2(t)\, dt$$

Power does not accumulate in time.

Energy : $E = \int_{-\infty}^{\infty} w^2(t)\, dt$ Watt seconds

no $\frac{1}{T}$ term.

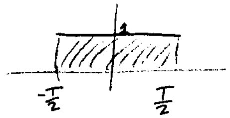
Energy signals : Energy is finite and nonzero
(Finite duration gives finite energy)

Power signals : Average power is finite and nonzero.
Must have infinite duration, which gives infinite energy

Power

$g(t) = 1$

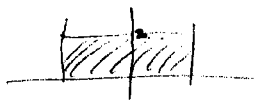
$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} (1)^2\, dt =$$



$$= 1$$

$g(t) = 2$

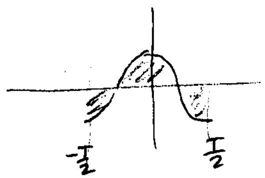
$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} (2)^2\, dt$$



$$= 4$$

confirms $P = V^2$

$g(t) = C\cos(\omega_0 t + \theta)$



$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} (C\cos(\omega_0 t + \theta))^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} C^2\cos^2(\omega_0 t + \theta)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \frac{C^2}{2}\left[1 + \cos(2\omega_0 t + 2\theta)\right] dt$$

$$= \underbrace{\lim_{T\to\infty} \frac{C^2}{2T}\int_{-\frac{T}{2}}^{\frac{T}{2}} 1\, dt}_{=1} + \underbrace{\lim_{T\to\infty} \frac{C^2}{2T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\omega_0 t + 2\theta)\, dt}_{=0}$$

$$= \frac{C^2}{2}$$

confirms $P = \frac{(V_{peak})^2}{2} = (V_{RMS})^2$

Decibel

used to express a power ratio.

$$dB = 10 \log_{10}\left(\frac{\text{one power level}}{\text{another power level}}\right)$$

Since $P = V^2, I^2, VI \ldots$

$$dB = 20 \log_{10}\left(\frac{\text{one voltage}}{\text{another voltage}}\right)$$

Power gain: $dB = 10 \log_{10}\left(\frac{P_{OUT}}{P_{IN}}\right)$

Voltage gain: $dB = 20 \log_{10}\left(\frac{V_{OUT}}{V_{IN}}\right)$

"Signal to noise" ratio --

$$(S/N)_{dB} = 10 \log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

$$= 20 \log_{10}\left(\frac{V_{RMS\ signal}}{V_{RMS\ noise}}\right)$$

Often, we will use dB relative to a reference:

dBm = dB relative to 1 milliwatt

$$dBm = 10 \log_{10}\left(\frac{P}{10^{-3}}\right)$$

$$dBW = 10 \log_{10}(P)$$

$$dBV = 20 \log_{10}(V)$$

$$dBmV = 20 \log_{10}\left(\frac{V}{10^{-3}}\right)$$

etc!

Signals & Vectors (2.5)

Signals are vectors.

Simple: vectors have magnitude and direction.

More complex: Space is n-dimensional.

Component of a vector -

We want to represent vectors by components:

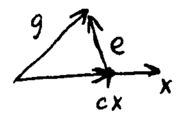
g is a vector —

x is another vector

signals context: g is one signal, x is another

We can make a projection of g onto x

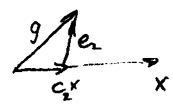
call it "cx" c is a constant



The vector from the end of the projection to the end of g is the "error vector".

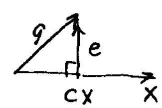
$$g = cx + e$$

We could have made a different projection:



$$g = c_2 x + e_2$$

Ideally, we choose c to minimize e.

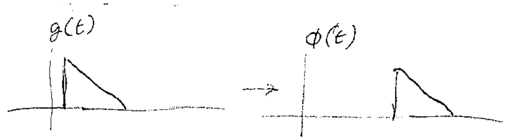


We would like to think that:

$$g \approx cx$$

2-3 - Operations on signals

Time shifting



$\phi(t+T) = g(t)$

or $\phi(t) = g(t-T)$

T = delay

Time shifting in a channel is inevitible.

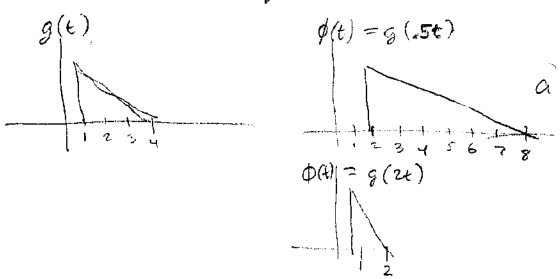
Time Scaling

Expansion or compression in time

$\phi(t) = g(at)$

$a<1 \rightarrow$ expanded in time (slowed down)

$a>1 \rightarrow$ compressed in time (speeded up)

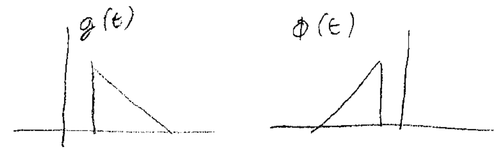


Expansion in time reduces bandwidth

Compression in time increases bandwidth

Channels usually don't do this, but we might do it deliberately.

Time inversion (Time reversal)



$\phi(-t) = g(t)$

or $\phi(t) = g(-t)$

Playing a signal backwards.

2.4 Unit impulse function

Defined as $\delta(t) = 0 \quad t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \qquad (1)$$

(implies it must be ∞ when $t = 0$)

practically – could think of it as a rectangular pulse, width $= \epsilon$, $\epsilon \to 0$

Multiplication of function by impulse

$$\phi(t)\,\delta(t) = \phi(0)\,\delta(t) \qquad (2)$$

Only the value at $t=0$ matters.

With time shifting —

$$\phi(t)\,\delta(t-T) = \phi(T)\,\delta(t-T) \qquad (3)$$

Only the value at $t=T$ matters.

Sampling –

$$\int_{-\infty}^{\infty} \phi(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} \phi(0)\,\delta(t)\,dt \qquad \text{from (2)}$$

$$= \phi(0) \int_{-\infty}^{\infty} \delta(t)\,dt$$

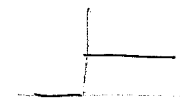
$$= \phi(0) \qquad \text{from (1)}$$

→ Area under the product of a function with an impulse equals the value of the function at that instant

$$\int_{-\infty}^{\infty} \phi(t)\,\delta(t-T)\,dt = \phi(T) \qquad \text{from (3)}$$

Unit step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



A signal is causal if it does not start before $t=0$

e^{-at} is non-causal

$e^{-at}u(t)$ is causal

$$\int_{-\infty}^{t} \delta(\tau)\,d\tau = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

integrating an impulse function gives us a unit step.

$$\frac{du}{dt} = \delta(t)$$

differenting a unit step gives us an impulse.