

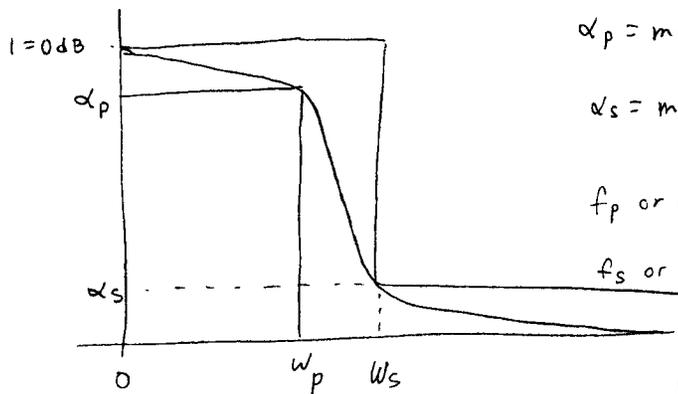
More analog --- Frequency Transformations ^{DSP-11C} ①

Basic idea ---

Instead of repeating it all for high pass, band pass, and band stop ---

- ① Transform the specification to low pass.
- ② Find the low pass transfer function.
- ③ Transform it back to the one you want.

Recall --- a low pass filter



α_p = max attenuation in passband
 α_s = min attenuation (loss) in stopband.

f_p or w_p = upper end of passband

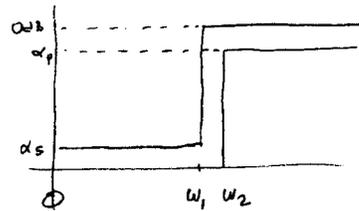
f_s or w_s = lower end of stopband.

$w_p < w_s$

Homework - no collect

- Stanley 6-11
- 7-12
- 7-13
- 7-14
- 7-15

High pass filters ②



Basis: Lowpass - pass $w_{LP} = 0$
 stop $w_{LP} = \infty$
 High pass - pass $w_{HP} = \infty$
 stop $w_{HP} = 0$

Transformation idea: substitute $w_{LP} = \frac{1}{w_{HP}}$, $s_{LP} = \frac{1}{s_{HP}}$

Method:

- ① Transform the specification to low pass:
 $w_p = \frac{1}{w_s}$
 $w_s = \frac{1}{w_p}$
- ② Design the low pass filter using methods already covered.
- ③ Transform it back

$s_{HP} = \frac{1}{s_{LP}}$

Example - A speaker crossover filter, for the tweeter

pass $f_2 > 3 \text{ kHz}$, within 3 dB, no ripple
 reject $f_1 < 1 \text{ kHz}$, at least 20 dB

Convert to radian frequency:

$$\text{pass } \omega_2 > 18850$$

$$\text{reject } \omega_1 < 6283$$

① Transform to low pass

$$\omega_c = \omega_p = \frac{1}{\omega_2} = \frac{1}{18850} = 5.305 \times 10^{-5} = 53.05 \times 10^{-6}$$

$$\omega_s = \frac{1}{\omega_1} = \frac{1}{6283} = 1.592 \times 10^{-4} = 159.2 \times 10^{-6}$$

② Design the low pass filter

$$2a - \text{How many poles} \quad N \geq \frac{\log_{10} \left[\frac{(10^{\frac{20}{N}} - 1)}{(10^{\frac{3}{N}} - 1)} \right]}{2 \log_{10} \left[\frac{159.2}{53.05} \right]} = \frac{\log_{10} [99/1.995]}{2 \log_{10} [3]}$$

$$= \frac{\log_{10} [99.47]}{2 (0.4771)} = \frac{1.998}{0.9542} = 2.093$$

Strictly, we need 3 poles, but sometimes you

can take less if it is close enough.

Use $N=2$.

2b - Design normalized filter

From table --

normalized low pass filter is:

$$H_n(s) = \frac{1}{s_n^2 + 1.414 s_n + 1}$$

2c - Transform the frequency (still low pass)

$$S_n = \frac{S_{LP}}{\omega_c} = \frac{S_{LP}}{53.05 \times 10^{-6}}$$

$$H_{LP}(S_{LP}) = \frac{1}{\left(\frac{S_{LP}}{53.05 \times 10^{-6}} \right)^2 + 1.414 \frac{S_{LP}}{53.05 \times 10^{-6}} + 1}$$

$$= \frac{1}{\frac{1}{2.8145 \times 10^{-9}} S_{LP}^2 + (26657) S_{LP} + 1}$$

It is easier if you leave it in this form.

③ Transform back to high pass

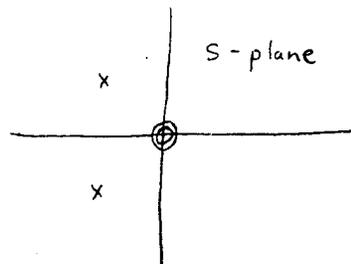
$$S_{HP} = \frac{1}{S_{LP}} \rightarrow S_{LP} = \frac{1}{S_{HP}}$$

$$H_{HP}(S_{HP}) = \frac{1}{\frac{1}{2.8145 \times 10^{-9}} \left(\frac{1}{S_{HP}} \right)^2 + 26657 \frac{1}{S_{HP}} + 1}$$

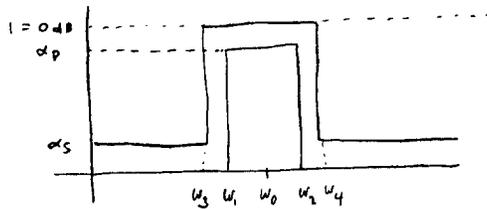
$$= \frac{S_{HP}^2}{\frac{1}{2.8145 \times 10^{-9}} + 26657 S_{HP} + S_{HP}^2}$$

$$H(S_{HP}) = \frac{S_{HP}^2}{\underbrace{355.31 \times 10^6}_{\text{This is } \omega_p^2} + 26657 S_{HP} + S_{HP}^2}$$

This is $1.414 \omega_p$



Band-pass (symmetrical)



$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$= \sqrt{\omega_3 \omega_4}$$

Basis: Shift the frequency, keep the "bandwidth".

$$s_{BP} = \frac{(s_{LP}^2 + \omega_0^2)}{s_{LP}}$$

Method:

① Transform the specification to low pass.

$$\omega_p = \omega_2 - \omega_1$$

← often ω_1 and ω_3 are given

$$\omega_s = \omega_4 - \omega_3$$

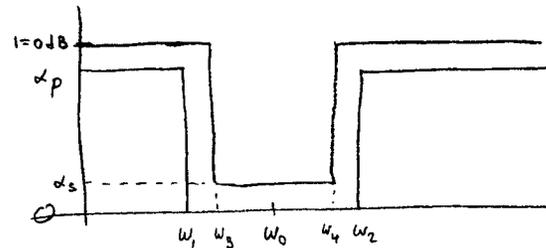
② Design the Low-pass filter.

③ Transform it back

$$s_{LP} = \frac{(s_{BP}^2 + \omega_0^2)}{s_{BP}}$$

⑤ Band reject (symmetrical)

⑥



Basis: shift the frequency, invert, like we did for high pass.
(or ... do both band pass and high pass transformations)

Method:

① Transform to low pass

$$\omega_p = \frac{1}{\omega_2 - \omega_1}$$

$$\omega_s = \frac{1}{\omega_4 - \omega_3}$$

② Design the low pass filter

③ Transform it back

$$s_{LP} = \frac{s_{BR}}{(s_{BR}^2 + \omega_0^2)}$$