

Table 6-3

Partially factored forms for low-pass filter denominator polynomials. (Refer to equation 6-36.)

Order	BUTTERWORTH	
1	(1 + s)	
2	(1 + 1.4142136s + s <sup>2</sup> )	
3	(1 + s + s <sup>2</sup> )	
4	(1 + 0.7653668s + s <sup>2</sup> ) (1 + 1.8477590s + s <sup>2</sup> )	
5	(1 + 0.6180340s + s <sup>2</sup> ) (1 + 1.6180340s + s <sup>2</sup> )	
CHEBYSHEV 0.5 DB RIPPLE		
1	(2.8627752 + s)	
2	(1.5162026 + 1.4256244s + s <sup>2</sup> )	
3	(0.6264565 + s) (1.1424477 + 0.6264564s + s <sup>2</sup> )	
4	(1.0635187 + 0.3507062s + s <sup>2</sup> ) (0.3564119 + 0.8466796s + s <sup>2</sup> )	
5	(0.3623196 + s) (1.0357841 + 0.2239258s + s <sup>2</sup> ) (0.4767669 + 0.5862454s + s <sup>2</sup> )	
CHEBYSHEV 1.0 DB RIPPLE		
1	(1.9652267 + s)	
2	(1.1025104 + 1.0977344s + s <sup>2</sup> )	
3	(0.4941706 + s) (0.9942046 + 0.4941706s + s <sup>2</sup> )	
4	(0.9865049 + 0.2790720s + s <sup>2</sup> ) (0.2793981 + 0.6737394s + s <sup>2</sup> )	
5	(0.2894933 + s) (0.9883149 + 0.1789168s + s <sup>2</sup> ) (0.4292978 + 0.4684100s + s <sup>2</sup> )	
CHEBYSHEV 2.0 DB RIPPLE		
1	(1.3075603 + s)	
2	(0.8230604 + 0.8038164s + s <sup>2</sup> )	
3	(0.3689108 + s) (0.8860951 + 0.3689108s + s <sup>2</sup> )	
4	(0.9286753 + 0.2097744s + s <sup>2</sup> ) (0.2215684 + 0.5064404s + s <sup>2</sup> )	
5	(0.218308 + s) (0.9521670 + 0.1349220s + s <sup>2</sup> ) (0.3931500 + 0.3532302s + s <sup>2</sup> )	
CHEBYSHEV 3.0 DB RIPPLE		
1	(1.0023773 + s)	
2	(0.7079478 + 0.6448996s + s <sup>2</sup> )	
3	(0.2986202 + s) (0.8391740 + 0.2986202s + s <sup>2</sup> )	
4	(0.9030867 + 0.1703408s + s <sup>2</sup> ) (0.1959800 + 0.4112390s + s <sup>2</sup> )	
5	(0.1775085 + s) (0.9360176 + 0.1097062s + s <sup>2</sup> ) (0.3770008 + 0.2872148s + s <sup>2</sup> )	

(These results are based on data obtained from L. A. Weinberg, *Network Analysis and Synthesis*, McGraw-Hill, 1962, with permission of author.)

tables is the failure to convert properly between radian and cyclic frequencies in scaling to the required frequency range.

The process of scaling the filter functions obtained from the table to the required frequency range consists of a linear scale change on the frequency axis. All frequencies are scaled in direct proportion. For example, if the scale change is such that the normalized cutoff of 1 rad/s is scaled to a cyclic frequency of 10 kHz, the original normalized response at 3 rad/s corresponds to the final scaled response at 30 kHz.

Let  $\omega_m$  represent some reference radian frequency in the normalized response, and assume that the desired corresponding reference radian frequency in the scaled response is to be  $\omega_r$ . Note that  $\omega_r$  may or may not be the cutoff frequency of the normalized response, as the scaling can center around any particular frequency desired. Let  $G(s)$  represent the normalized transfer function, and let  $G_1(s)$  represent the

Table 6-2

Coefficients of low-pass filter denominator polynomials. (Note: Coefficients are defined in accordance with equation 6-35.)

Order	B <sub>0</sub>	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>	B <sub>5</sub>
BUTTERWORTH						
1	1					
2	1	1.4142136				
3	1	2				
4	1	2.6131259	3.4142136	2.6131259		
5	1	3.2360680	5.2360680	5.2360680	3.2360680	1
CHEBYSHEV 0.5 DB RIPPLE ( $\epsilon^2 = 0.1220184$ )						
1	2.8627752					
2	1.5162026	1.4256245				
3	0.7156938	1.5348954	1.2529130			
4	0.3790506	1.0254553	1.7168662	1.1973856		
5	0.1789234	0.7525181	1.3095747	1.9373675	1.1724909	1
CHEBYSHEV 1 DB RIPPLE ( $\epsilon^2 = 0.2589254$ )						
1	1.9652267					
2	1.1025103	1.0977343				
3	0.4913067	1.2384092	0.9883412			
4	0.2790567	0.7426194	1.4539248	0.9528114		
5	0.1228267	0.5805342	0.9743961	1.6888160	0.9368201	1
CHEBYSHEV 2 DB RIPPLE ( $\epsilon^2 = 0.5848932$ )						
1	1.3075603					
2	0.8230604	0.8038164				
3	0.3268901	1.0221903	0.7378216			
4	0.2057651	0.5167981	1.2564819	0.7162150		
5	0.0817225	0.4593491	0.6934770	1.4995433	0.7064606	1
CHEBYSHEV 3 DB RIPPLE ( $\epsilon^2 = 0.9952623$ )						
1	1.0023773					
2	0.7079478	0.6448996				
3	0.2505943	0.9283480	0.5972404			
4	0.1769869	0.4047679	1.1691176	0.5815799		
5	0.0626391	0.4079421	0.5488626	1.4149847	0.5744296	1

(These results were obtained from L. A. Weinberg, *Network Analysis and Synthesis*, McGraw-Hill, 1962, with permission of the author.)

The choice for  $A_0$  follows the same logic discussed in the preceding paragraph.

In using the tables, it is important to remember that these functions are normalized with respect to a radian frequency of  $\omega_c = 1$  rad/s, whereas most specifications are given in terms of cyclic frequency. As obvious as this may seem, a very common source of error in using such