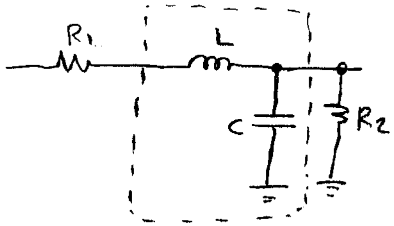
A closer look at a second order filter

(1)

Why? — Any higher order filter can be made of 1st and 2nd order sections because poles are in conjugate pairs.



Filter.

Analysis:

Voltage divider approach!

$$\frac{Z_{shunt}}{Z_{series}+Z_{shunt}} = \frac{\frac{1}{\frac{1}{R_2}+sC}}{R_1+sL+\frac{1}{\frac{1}{R_2}+sC}} = \frac{1}{R_1\left(\frac{1}{R_2}+sC\right)+sL\left(\frac{1}{R_2}+sC\right)+1}$$

$$= \frac{1}{\frac{R_1}{R_2}+sCR_1+s\frac{L}{R_2}+s^2LC+1} = \frac{1}{s^2(LC)+s\left(CR_1+\frac{L}{R_2}\right)+\frac{R_1}{R_2}+1}$$

$$= \frac{R_2}{s^2(LCR_2)+s(CR_1R_2+L)+R_1+R_2} = \frac{\frac{R_2}{R_1+R_2}}{s^2\underbrace{\left(\frac{LCR_2}{R_1+R_2}\right)}_{\frac{1}{\omega_0^2}}+s\underbrace{\left(\frac{CR_1R_2+L}{R_1+R_2}\right)}_{\frac{1}{\omega Q}=\frac{D}{\omega}}+1}$$

Check: Low pass? yes.

Correct gain? $= \frac{R_2}{R_1+R_2}$ yes.

Powers of s match # of reactive elements? yes.

Example! $f = 1\text{ KHz}$ $D = 1.414$ ← You are given this specification.

$R_1 = R_2 = 50\,\Omega$.

(2)

↑ "maximally flat" "Butterworth" more to come.

$$\omega = 2\pi f = 6283$$

Two equations:

$$\frac{1}{\omega_0^2} = \frac{LCR_2}{R_1+R_2} \qquad \frac{D}{\omega} = \frac{CR_1R_2+L}{R_1+R_2}$$

$$\frac{1}{(6283)^2} = LC(.5) \qquad \frac{1.414}{6283} = C(50)(.5)+\frac{L}{100}$$

$$LC = \frac{2}{6283^2} = \frac{2}{39.48\times10^6} = 50.66\times10^{-9} \qquad \frac{1.414}{6283} = 25C+\frac{L}{100}$$

$$\frac{141.4}{6283} - 2500C = L$$

$$.0225 - 2500C = L$$

$$(.0225-2500C)C = 50.66\times10^{-9}$$

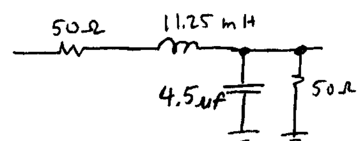
$$.0225C - 2500C^2 = 50.66\times10^{-9}$$

$$2500C^2 - .0225C + 50.66\times10^{-9} = 0$$

$$C = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{.0225\pm\sqrt{506.3\times10^{-6}-(4)(2500)(50.66\times10^{-9})}}{2(2500)}$$

$$= \frac{.0225}{5000} = 4.5\times10^{-6}$$

$$L = .0225 - 2500C = .01125 = 11.25\times10^{-3}$$



(checked with simulator −9 dB, −90° @ 1KHz) 40dB/dec

Alternative design technique —

Design for $R = 1\,\Omega$ $\quad \omega = 1$

Then scale it.

Why? ① It's easier.

② Often tables are available.

$$\frac{1}{\omega_0^2} = \frac{LCR_2}{R_1+R_2} \qquad \frac{D}{\omega} = \frac{CR_1R_2 + L}{R_1+R_2}$$

$$1 = .5LC \qquad 1.414 = \frac{C+L}{2}$$

$$LC = 2 \qquad 2.828 = C+L$$

$$2.828 - C = L$$

$$(2.828 - C)C = 2$$

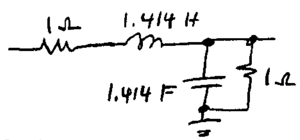
$$2.828C - C^2 = 2$$

$$C^2 - 2.828C + 2 = 0$$

$$C = \frac{2.828 \pm \sqrt{8-8}}{2} = \frac{2.828}{2} = 1.414$$

$$L = 2.828 - C = 1.414$$

Normalized filter.

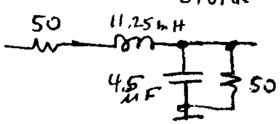


Scale Frequency — divide L and C by ω

$$\frac{1.414}{6283} = \begin{matrix} 225\,\mu F \\ 225\,\mu H \end{matrix}$$

Scale resistance multiply L by R : $(225\,\mu H)(50) = 11.25\,mH$

divide C by R : $\frac{(225\,\mu F)}{50} = 4.5\,\mu F$

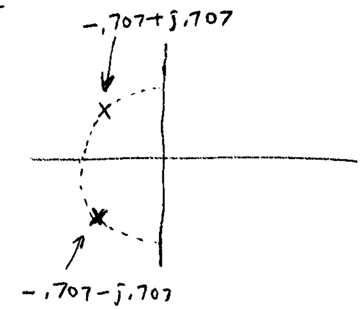


Poles + zeros of the normalized filter:

$$\frac{\frac{R_2}{R_1+R_2}}{s^2\left(\frac{LCR_2}{R_1+R_2}\right) + s\left(\frac{CR_1R_2 + L}{R_1+R_2}\right) + 1}$$

$$= \frac{\frac{1}{1+1}}{s^2\left(\frac{(1.414)(1.414)(1)}{1+1}\right) + s\left(\frac{(1.414)(1)(1) + 1.414}{1+1}\right) + 1}$$

$$= \frac{.5}{s^2 + 1.414s + 1}$$



Active filters

Want to eliminate inductors.

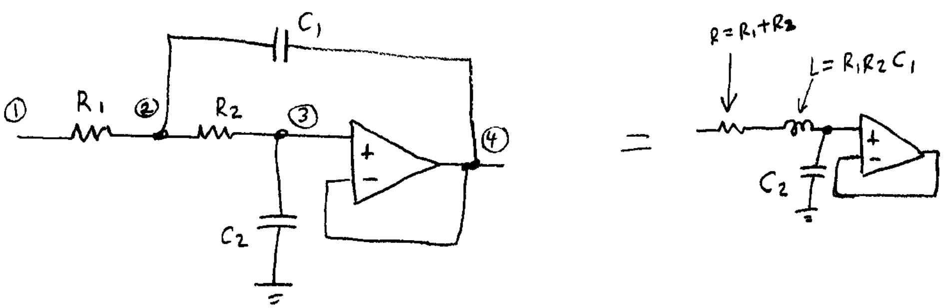
Two strategies: (1) Mimic the inductor – "Gyrator" approach.

(2) solve the differential equation.

Strategy (1) Mimic the inductor.

"Sallen + Key" design

↑ the inventors.



Analysis:

Node equations

(2) $\frac{1}{R_1}(V_2-V_1) + \frac{1}{R_2}(V_2-V_3) + sC_1(V_2-V_4) = 0$

(3) $\frac{1}{R_2}(V_3-V_2) + sC_2(V_3) = 0$

(4) $V_3 = V_4$

Group V terms..

(2) $V_1\left(-\frac{1}{R_1}\right) + V_2\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1\right) + V_3\left(-\frac{1}{R_2}\right) + V_4(-sC_1) = 0$

(3) $V_2\left(-\frac{1}{R_2}\right) + V_3\left(\frac{1}{R_2}+sC_2\right) = 0$

Solve (3) for V_2

$$V_2 = V_3\left(\frac{1}{R_2}+sC_2\right)(R_2)$$

$$= V_3(1+sC_2R_2)$$

Subst $V_3 = V_4$ $\quad V_2 = V_4(1+sC_2R_2)$

sub in (2)

$$V_1\left(-\frac{1}{R_1}\right) + V_4(1+sC_2R_2)\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1\right) + V_4\left(-\frac{1}{R_2}\right) + V_4(-sC_1) = 0$$

clean up.

$$V_1\left(-\frac{1}{R_1}\right) + V_4\left[\frac{1}{R_1} + \cancel{\frac{1}{R_2}} + \cancel{sC_1} + \frac{sC_2R_2}{R_1} + \frac{sC_2\cancel{R_2}}{\cancel{R_2}} + (sC_2R_2)(sC_1) - \cancel{\frac{1}{R_2}} - \cancel{sC_1}\right] = $$

$$V_1\left(-\frac{1}{R_1}\right) + V_4\left[s^2(C_1C_2R_2) + s\left(\frac{C_2R_2}{R_1}+C_2\right) + \frac{1}{R_1}\right] = 0$$

mult by R_1

$$V_1(-1) + V_4\left[s^2(C_1C_2R_1R_2) + s(C_2R_2+C_2R_1) + 1\right] = 0$$

$$\frac{V_4}{V_1} = \frac{1}{s^2(C_1C_2R_1R_2) + s(C_2R_2+C_2R_1) + 1}$$

Check: Low pass? yes.

Correct gain? yes.

Powers of s match? yes.

Design a filter—

"Normalized" $R=1$, $\omega=1$. $D=1.414$.

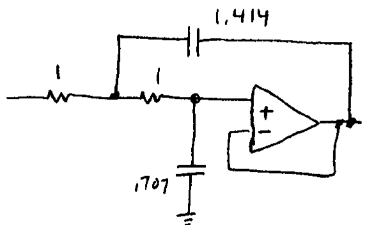
$$\frac{1}{s^2+1.414s+1}$$

$$\frac{1}{\omega_o^2} = C_1C_2R_1R_2 \qquad \frac{D}{\omega} = C_2(R_1+R_2)$$

$$C_1C_2 = 1 \qquad 2C_2 = D = 1.414$$

$$C_2 = \frac{1.414}{2} = .707$$

$$C_1 = \frac{1}{C_2} = \frac{1}{.707} = 1.414$$



Design for 1 KHz, $D = 1.414$, $R_1 = R_2 = 10K$.

Scale f: divide C by $\omega = 2\pi f$

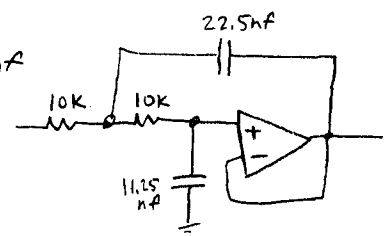
$$C_1 = \frac{1.414}{6283} = 225\,\mu f$$

$$C_2 = \frac{.707}{6283} = 112.5\,\mu f$$

Scale R: divide C by R

$$C_1 = \frac{225\times10^{-6}}{10^5} = 22.5\text{ nf}$$

$$C_2 = \frac{112.5\times10^{-6}}{10^5} = 11.25\text{ nf}$$



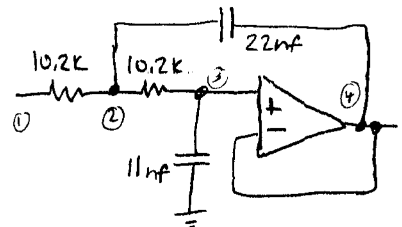
These capacitors have ugly values.

Use 22 nf and 11 nf (standard 5% values)

Change R: We lowered both C's by .978

so increase R to 10.23K.

10.2K is a standard 1% value.



10K has actual corner at 1.03 KHz.

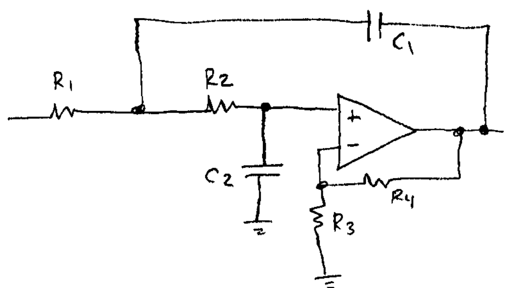
dB error at 1k = .2 dB

close enough.

Filters often use precision components.

(1% tolerance)

More general Low pass active filter



R3, R4 set gain
also damping.

$$K = 1 + \frac{R_4}{R_3}$$

$$\frac{V_{out}}{V_{in}} = \frac{K}{s^2(R_1R_2C_1C_2) + s\left(R_1C_2 + R_2C_2 + R_1C_1(1-K)\right) + 1}$$

Good values:

$$\frac{K}{s^2\left(\frac{1}{\omega^2}\right) + s\left(\frac{d}{\omega}\right) + 1}$$

① "Normalized" filter $\omega = 1$, $K = 1$

$R_1 = 1$ $R_2 = 1$ $C_1 = \frac{2}{d}$ $C_2 = \frac{d}{2}$

$$\frac{K\omega}{s^2 + s(d\omega) + \omega^2}$$

② Equal component value $\omega = 1$

$R_1 = 1$ $R_2 = 1$ $C_1 = 1$ $C_2 = 1$

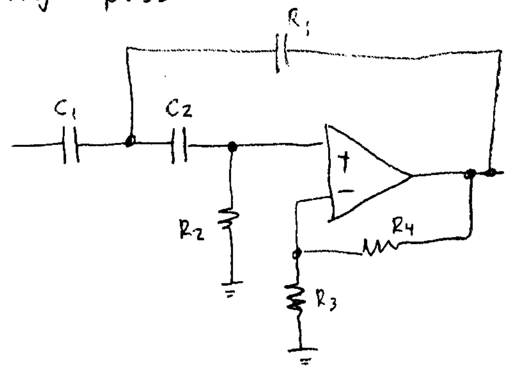
$R_3 = 1$ $R_4 = 2-d$ $K = 3-d$

Then: scale frequency: $C = \frac{C_N}{\omega}$

scale resistance: $C = \frac{C_{(R=1)}}{R}$ Keep R_1, R_2 equal.

(RC = constant)

High pass



$$\frac{V_{out}}{V_{in}} = \frac{K\, s^2(R_1R_2C_1C_2)}{s^2(R_1R_2C_1C_2) + s\left(R_1C_2 + R_1C_1 + R_2C_2(1-K)\right) + 1}$$

Good values:

$$\frac{K\, s^2/\omega^2}{s^2/\omega^2 + s(d/\omega) + 1}$$

① Normalized, unity gain;

$\omega = 1$ $K = 1$

$C_1 = 1$ $C_2 = 1$ $R_1 = \frac{d}{2}$ $R_2 = \frac{2}{d}$

$$\frac{Ks^2}{s^2 + s(d\omega) + \omega^2}$$

② Equal component value

$C_1 = 1$ $C_2 = 1$ $R_1 = 1$ $R_2 = 1$

$R_3 = 1$ $R_4 = 2-d$ $K = 3-d$

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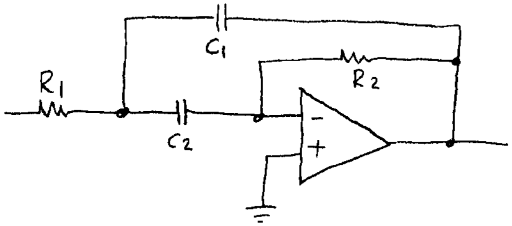
Band pass – multiple feed back (Low Q)



$$\frac{V_{out}}{V_{in}} = \frac{-s(R_2C_2)}{s^2(R_1R_2C_1C_2) + s(R_1C_2 + R_1C_1) + 1}$$

$$\frac{Ks}{s^2/\omega^2 + s(d\omega) + 1}$$

Good values: $(\omega = 1)$

$$\frac{Ks/\omega^2}{s^2 + s(d\omega) + \omega^2}$$

$C_1 = 1 \quad C_2 = 1 \quad R_1 = \frac{1}{2Q} \quad R_2 = 2Q$

$(R_1R_2 = 1)$

$d = \frac{1}{Q}$

Gain at $s^2 = -1$ $(s = j)$:

$$\frac{-j(R_2)}{\underbrace{-1(R_1R_2)}_{-1} + j(R_1 + R_1) + 1} = \frac{-R_2}{2R_1}$$

$$\frac{-j(2Q)}{-1 + j(2/2Q) + 1} = \frac{-2Q}{\frac{2}{2Q}} = \frac{-4Q^2}{2} = -2Q^2$$

Gain and Q are related.

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Example: $f = 1\,kHz$ bandwidth $= 100\,Hz$

$Q = 10$

Normalized.

$C_1 = 1 \quad C_2 = 1 \quad R_1 = \frac{1}{20} \quad R_2 = 20 \quad R_1R_2 = 1$

Gain $= -2Q^2 = \underline{\underline{-200}}$

Scale Frequency

$C_1 = C_2 = \frac{1}{6283}$

Scale to $R_1 = 1K$, then $R_2 = 400K$

$R_1R_2 = 400 \times 10^6$

$\sqrt{R_1R_2} = 20\,K$

$$C_1 = C_2 = \frac{1}{(6283)(20K)} = 7.958\ nf$$

Actually use $C_1 = C_2 = 10\ nf$

$R_1 = 7.958\ K$ $\quad \left\{ \begin{array}{l} 7.87\,k \\ 8.05\,k \leftarrow \end{array} \right.$

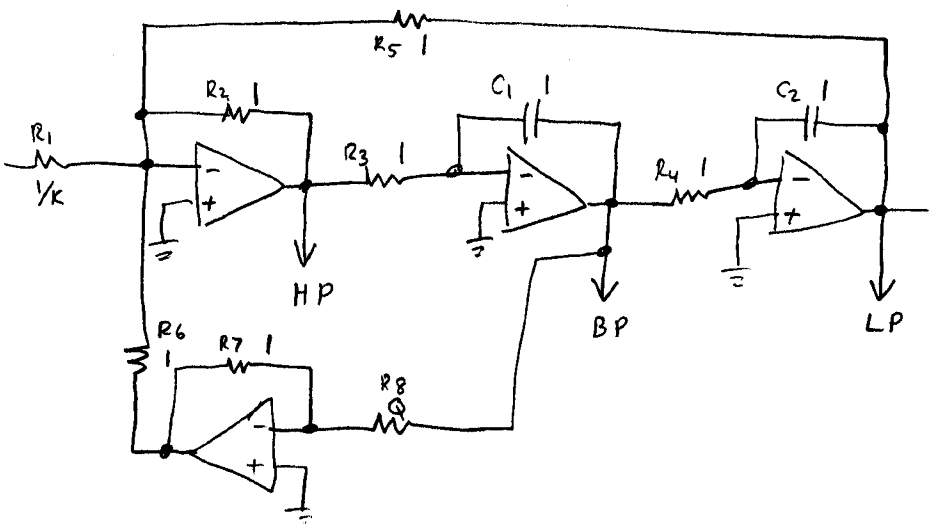
$R_2 = 318.3\ K$ $\quad \left\{ \begin{array}{l} 316\,k \leftarrow \\ 323\,k \end{array} \right.$

1% std values

$\sqrt{R_1R_2} = 50.33\,K$

$\sqrt{R_1R_2} = 50.44\,K$

4 op-amp filter (high Q)



~~Good values~~

$$Q = \frac{R_8}{R_7} \qquad W = \frac{1}{R_3 R_4 C_1 C_2}$$

Good values :

$R_3 = R_4 \qquad C_1 = C_2$

$R_2 = R_5 = R_6$

$R_1 = \frac{R_2}{K}$