Analog filters - basic concepts.

Basic goal — pass some signals, reject others.

Make the selection based on frequency.

Types:

Low pass High pass Band pass Band stop

Uses:

Low pass — remove artifacts of sampling (CD's, FM stereo, radio, TV)

protect digital filter from aliasing

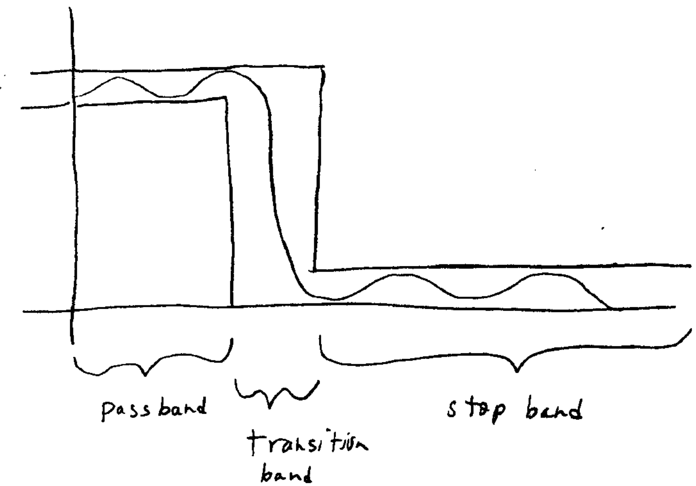
High pass — remove low frequency noise (turntable rumble)

Band pass — Select a communication channel (radio, TV)

Band stop — Eliminate a particular interefering signal (usually done as a notch)

All pass — Time delays or phase shift.

Filter specification



→ Gain or loss in pass band must be between certain limits.

Example - audio FM "20 Hz to 15 kHz $^{+0}_{-1}$ dB"

(no loss to 1 dB loss in pass band)

→ Loss in pass band must be at least a certain amount.

Example: loss must be at least 60 dB for f > 18 kHz

So, for this filter, the pass band is 20 Hz to 15 KHz.

It looks like band pass, but there is no loss specified for less than 20 Hz, so call it low pass -- 0 Hz to 15 kHz.

The stop band is 18 KHz to infinity.

The transition band is 15 kHz to 18 KHz.

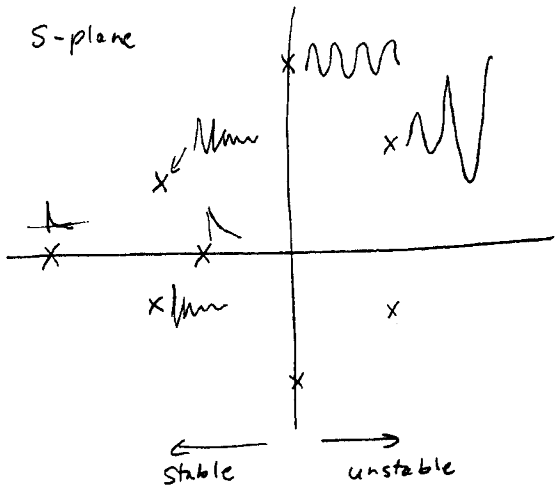
Laplace transform (not covering in detail)

converts time domain to "s" domain.

$$X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt$$

$$x(t) = \frac{1}{2\pi j} \int_c X(s)\, e^{st}\, ds$$

For frequency response, let $s = j\omega$.



Second order system on s-plane

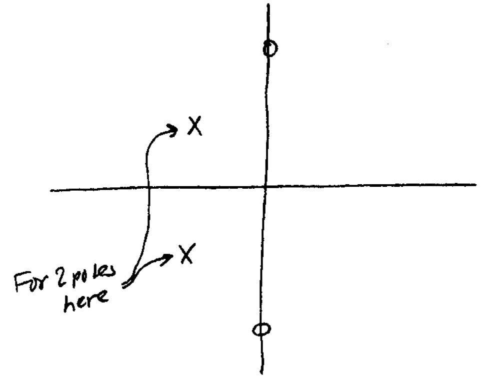
← response based on pole locations.

Form is

$$X(s) = \frac{N(s)}{D(s)} = \frac{s^2 + as + b}{s^2 + cs + d}$$

Poles are roots of denominator

Zeros are roots of numerator



Imagine a stretched membrane —

Tacked to 0 at the zeros -

Pulled up to infinity at the poles.

Travel along $j\omega$ to get frequency response.

Zeros on $j\omega$ axis → frequencies of zero response.

So...

$$H(s) = \frac{1}{s^2 + cs + d}$$

Roots of numerator:
$s = \infty, \infty$
No transmission at infinity
→ Low pass

$$H(s) = \frac{s^2}{s^2 + cs + d}$$

Roots of numerator:
$s = 0, 0$
No transmission at zero
→ High pass

$$H(s) = \frac{s}{s^2 + cs + d}$$

Roots of numerator:
$s = 0, \infty$
No transmission at zero or infinity
→ band pass

There are always the same number of poles and zeros. The ones that appear to be missing are at infinity.

Note similarity in form —

The denominators are the same.

The numerators determine the type.

Real filters can have any order.
(power of s in the polynomial).

When there are two or more poles,
they are in conjugate pairs.

A common design technique is to first design a low-pass filter then transform it to high pass or whatever.

Another common design technique is to lookup in tables for normalized prototype filters (1 Hz bandwidth) and transform to actual bandwidth desired.

Frequency response in analog domain.

Let $s = jw$ and sweep.

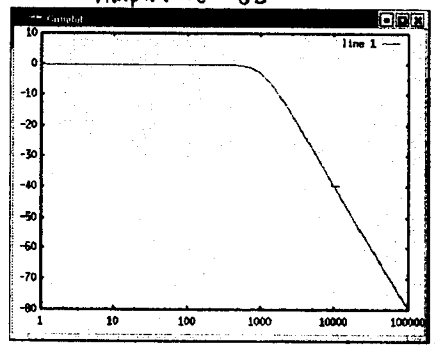
Example:

$$H(s) = \frac{1000000}{s^2 + 1414s + 1000000}$$

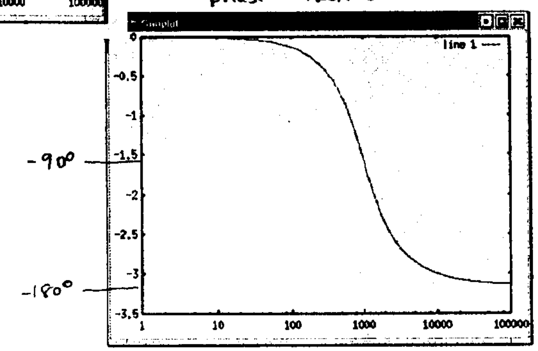
Let $s = jw$ and sweep

$$H(jw) = \frac{1000000}{(jw)^2 + 1414jw + 1000000}$$

Amplitude dB



phase - radians



This should be review from signals + systems.
"Bode plot"

But ...

I want to do it the otherway.

Given a frequency response –
Develop a transfer function
that approximates it.

Example: $\omega_c = 1000$ radians

Loss = +0, −3 db in passband

Loss > 40 dB for $\omega > 10000$

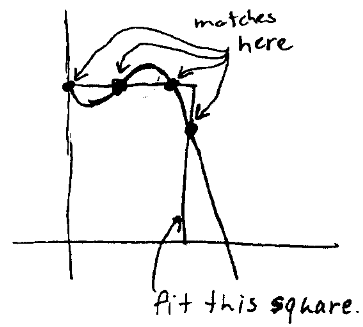
-- is a spec that is met by this filter.

Approximation methods:

Reference: polynomial fitting from Numerical Analysis.

Idea:

Given a shape, we can fit a polynomial to it.



We can make an interpolating polynomial that matches at some points.