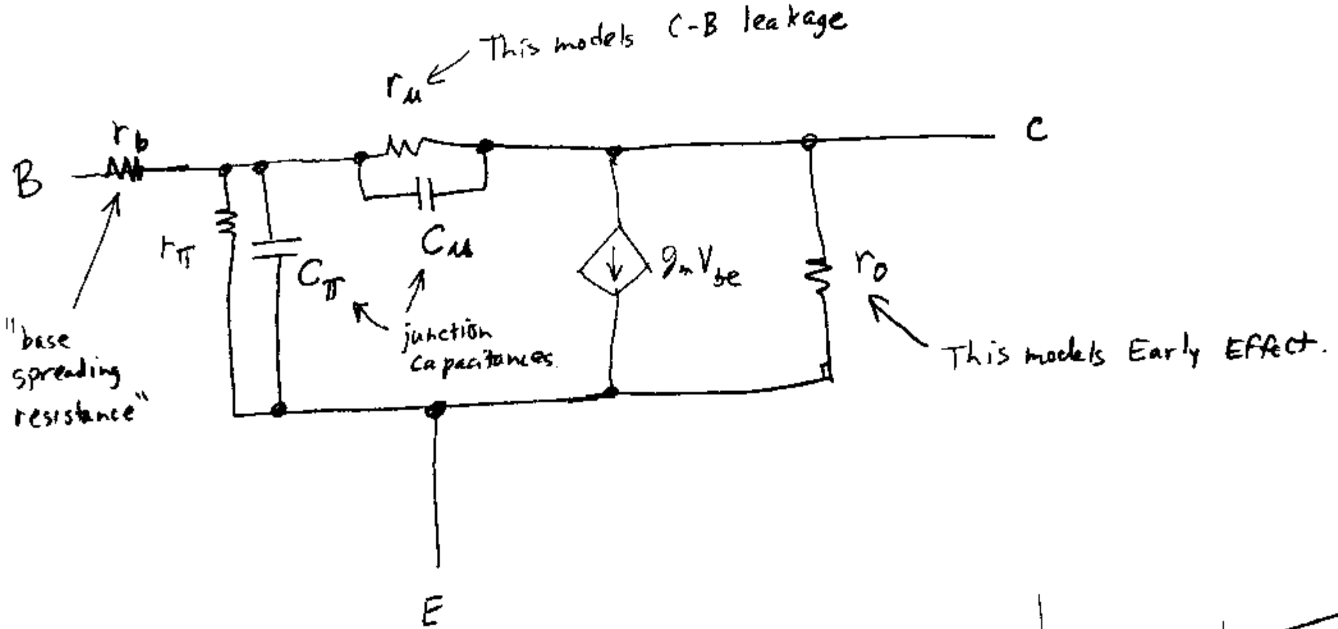
A better small-signal model..

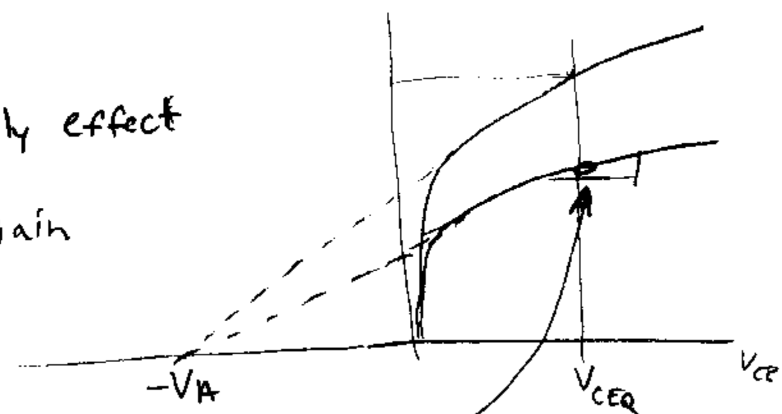
The "hybrid-pi" model..

Add some more complexity to our model ---



r_o = resistance due to Early effect

Important – it lowers gain



Calculate from curves--

$r_o = \frac{\Delta V}{\Delta I}$ on collector curve

or from V_A --

$$r_o = \frac{V_{CEQ} + V_A}{I_{CEQ}}$$

Example:

$V_{CEQ} = 10V$, $V_A = 90V$.

$I_{CEQ} = 1ma$

$$r_o = \frac{100V}{1ma} = 100K$$

C_π = base-emitter capacitance
Spice parameter Cje

C_μ = base-collector capacitance
Spice parameter Cjc

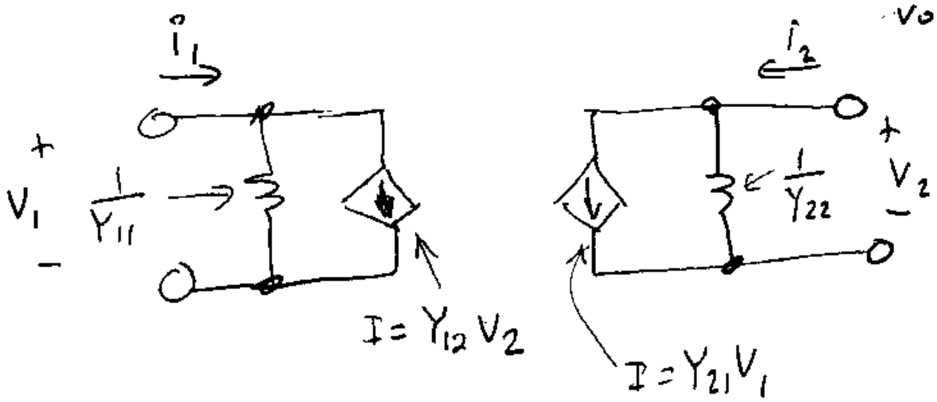
r_μ = "reverse-biased diffusion resistance"
similar to r_π but for C-B junction
since it is reverse biased, it is very high.

r_b = "base spreading resistance"
usually $\approx 10\,\Omega$
usually you can ignore it
(compare to $r_\pi \approx 2600\,\Omega$)

We can express any "2-port" using standardized parameters.

Simplest first ... "Y" parameters (admittance)

All are in the form of $\frac{\text{current}}{\text{voltage}}$



Large signal

$$Y_{11} = \frac{I_1}{V_1} \qquad Y_{21} = \frac{I_2}{V_1}$$

$$Y_{12} = \frac{I_1}{V_2} \qquad Y_{22} = \frac{I_2}{V_2}$$

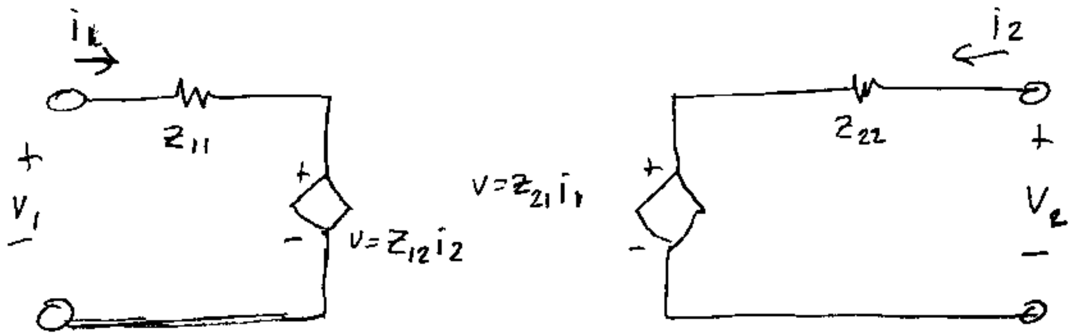
Small signal

$$y_{11} = \frac{i_1}{v_1} \qquad y_{21} = \frac{i_2}{v_1}$$

$$y_{12} = \frac{i_1}{v_2} \qquad y_{22} = \frac{i_2}{v_2}$$

There are others ...

Z parameters = impedance = $\frac{\text{volts}}{\text{amps}}$.



By Thevinin / norton equivalent circuits —

You can translate between Y and Z parameters —

$$y_{11} = \frac{i_1}{V_1}, \quad z_{11} = \frac{V_1}{i_1} \quad \text{so} \quad y_{11} = \frac{1}{z_{11}} \quad z_{11} = \frac{1}{y_{11}}$$

$$y_{22} = \frac{i_2}{V_2} \quad z_{22} = \frac{V_2}{i_2} \quad \text{so} \quad y_{22} = \frac{1}{z_{22}} \quad z_{22} = \frac{1}{y_{22}}$$

$$y_{21} = \frac{i_2}{V_1} \quad z_{21} = \frac{V_2}{i_1}$$

Let $V_1 = \frac{i_1}{y_{11}}$, $i_2 = y_{22} V_2$

substitute

$$Y_{21} = \frac{y_{22} V_2}{\frac{i_1}{y_{11}}} = y_{22} y_{11} \frac{V_2}{i_1}$$

$$\frac{y_{21}}{y_{22} y_{11}} = \frac{V_2}{i_1} \quad \therefore \quad z_{21} = \frac{y_{21}}{y_{22} y_{11}}$$

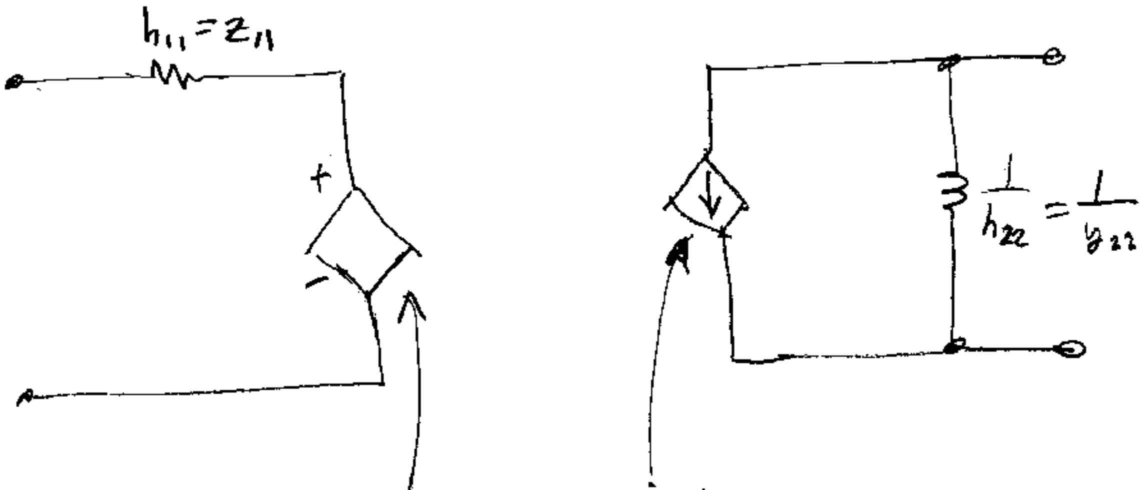
Let $V_2 = z_{22} i_2$, $i_1 = \frac{V_1}{z_{11}}$

$$z_{21} = \frac{z_{22} i_2}{\frac{V_1}{z_{11}}} = z_{11} z_{22} \frac{i_2}{V_1} = z_{11} z_{22} y_{21}$$

$$y_{21} = \frac{z_{21}}{z_{11} z_{22}}$$

H means Hybrid.

Make it like Z parameters on input, Y on output.



$v = z_{12} i_2$

$v = h_{12} v_2$

$i = y_{21} v_1$

$i = h_{21} i_1$

For transistors, port 1 is B-E
port 2 is C-E
↑ Common emitter.

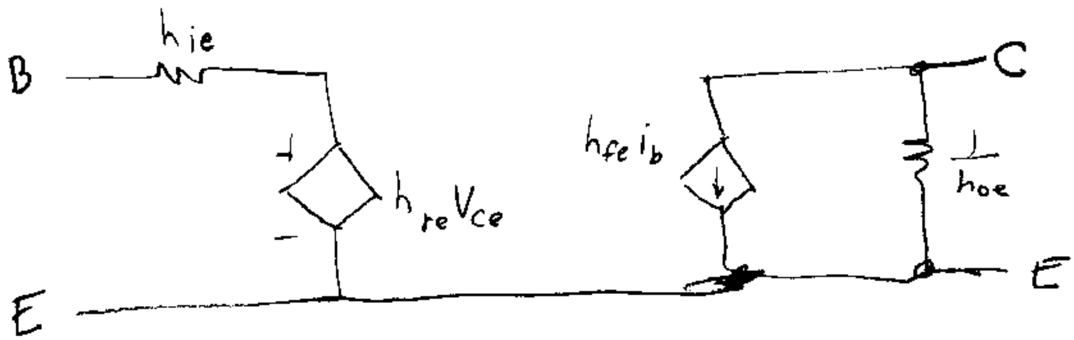
We have another set of names.

$h_{ie} = h_{11}$ — ie means "input, common emitter"

$h_{re} = h_{12}$ re -- "reverse, common emitter"

$h_{fe} = h_{21}$ fe -- "forward, common emitter"

$h_{oe} = h_{22}$ oe -- "output, common emitter"



So, if we ignore the capacitances and r_μ ...

$$h_{ie} = r_\pi$$
$$h_{fe} = \beta$$
$$h_{oe} = \frac{1}{r_o}$$
$$h_{re} = \frac{r_\pi}{r_\pi + r_\mu}$$

The book uses UPPER case letters when it should use lower here.
These are smallsignal parameters.

See graphs – p. 183 of text.

HW –

	P	#
	177-178	1,2
	179	3,4
	230	1,4

Example: $\beta = 100$, $V_A = 90$, $I_{CQ} = 1\text{ma}$, $V_{BE} = 10$

$$g_m = \frac{\partial i_c}{\partial v_{be}} = \frac{I_{CQ}}{V_T} = \frac{1\text{ma}}{.026} = .038\ \frac{A}{V} \qquad \left(38\ \frac{\text{ma}}{V}\right)$$

$$r_\pi = \frac{\partial v_{be}}{\partial i_b} = \frac{.026}{I_B} = \frac{.026}{.01\text{ma}} = 2.6\text{K} = 2600\,\Omega$$

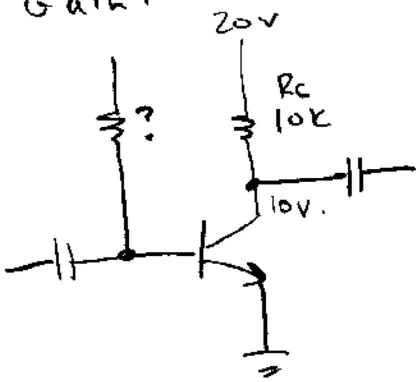
$$r_o = \frac{\partial v_{ce}}{\partial i_c} = \frac{V_A + V_{CE}}{I_{CQ}} = \frac{90+10}{1\text{ma}} = \frac{100}{1\text{ma}} = 100\text{K}$$

$h_{ie} = r_\pi = 2600\,\Omega$ $\qquad$ $h_{fe} = 100$ $\quad (= \beta)$

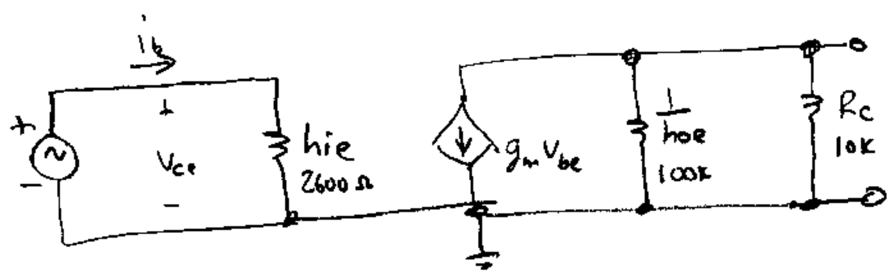
$h_{re} = 0$ (no info given) $\qquad$ $h_{oe} = \frac{1}{100\text{K}} = 10^{-5}$

Gain:

(Transistor as above)



Equivalent circuit:



Gain $= g_m R_L$ $\qquad$ where $R_L = \frac{1}{h_{oe}} \parallel R_C$

$= 9\text{K}$

Gain $= (.038)(9000) = 342$

(would be 380 if we ignored r_o)