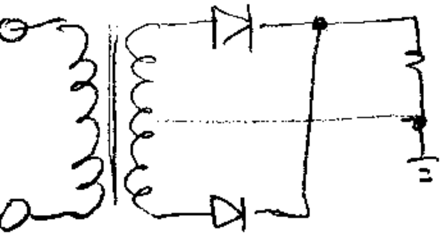
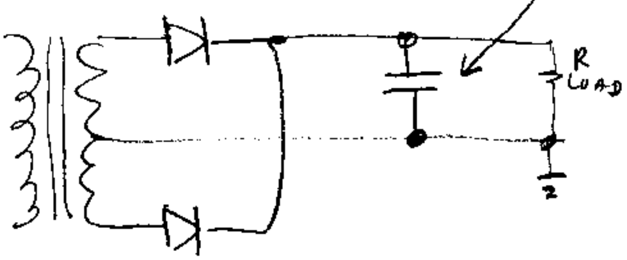
Power supply filters (2.1.3)

Recall ...
A rectifier puts out something like this:

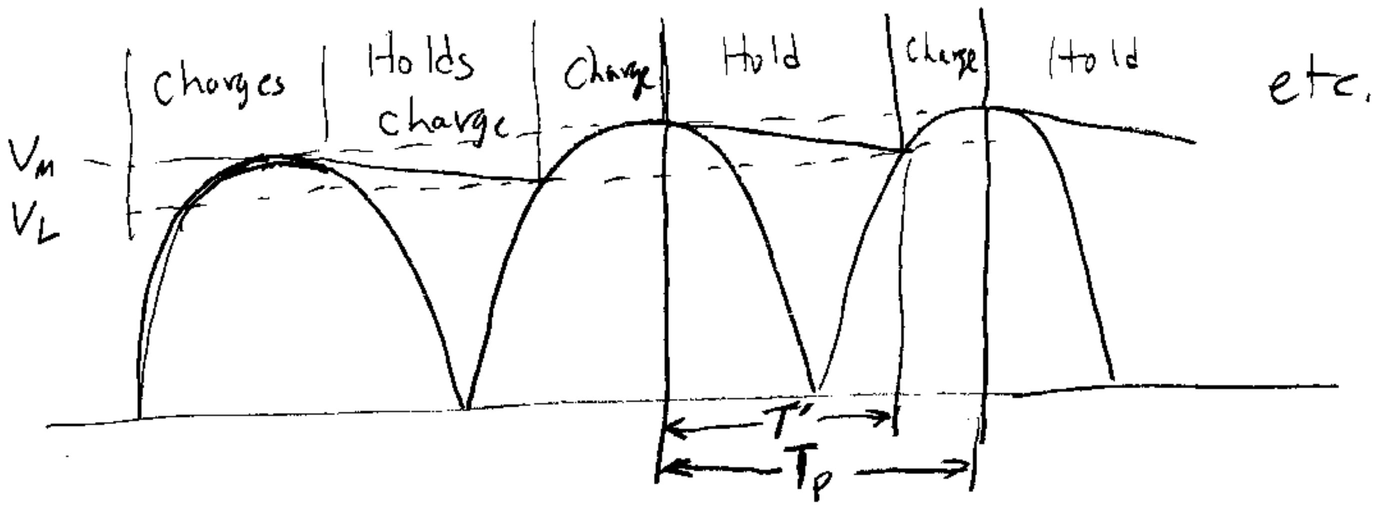


We want DC, without ripple.

Idea: Add a capacitor



It charges when the diode is on, then holds the charge.



The capacitor holds the voltage to nearly the peak voltage.

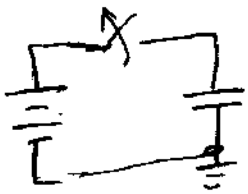
The diode is only on for a short period at the peak (marked "Charge" here).
The output has a small (?) ripple.

Recall -- The formula for a capacitor voltage decaying --

$$V_o(t) = V_M e^{\frac{-t}{\tau}}$$

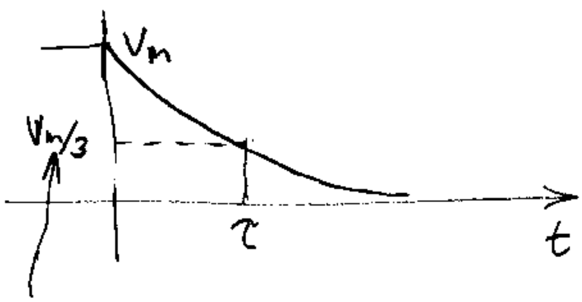
t = time s/n

τ = time constant
$= RC$



← Switch opens at $t=0$

(It is like the input dropping so the diode turns off)



approximately

(Actually -- $.368 V_M$

$.368 = e^{-1} \approx \frac{1}{3}$)

For power supply filters, we can only tolerate a little ripple, so τ is much longer than the ripple period. (T_p)
(time between peaks)

The smallest output voltage is when the diode turns back on -- after time T'

$$V_L = V_M e^{\frac{-T'}{\tau}}$$

Ripple voltage is the difference --

$$V_r = V_M - V_L = V_M \left(1 - e^{-\frac{T'}{\tau}}\right)$$

Recall ...

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

(Taylor series)

If x is small, $1 + x$ is a good approximation.

so...

$$e^{-\frac{T'}{\tau}} \approx 1 - \frac{T'}{\tau}$$

$$V_r \approx V_M \left(1 - \left(1 - \frac{T'}{\tau}\right)\right)$$

$$= V_M \left(\frac{T'}{\tau}\right)$$

T' is a little less than T_p.

and hard to solve

Substitute T_p for T' -

The error is small and in the safe direction.

so (also sub. $\tau = RC$)

$$V_r \approx V_M \left(\frac{T_p}{RC}\right)$$

Actual ripple will be a little less.
This is close enough.

For half wave rectifier

$$T_p = \frac{1}{f}$$

f is the AC "signal?" frequency.

For full wave rectifier

$$T_p = \frac{1}{2f}$$

Example:

A power supply must deliver 100 volt at 100 ma with less than 10 volts (peak to peak) ripple. What is C? for half wave? full wave?

Power frequency = 60 Hz.

Solution: $R_{LOAD} = \frac{100\,v}{.1\,A} = 1000\,\Omega$

$$V_r \approx V_m\left(\frac{T_P}{RC}\right) \Rightarrow C \approx \frac{V_m}{V_r}\frac{T_P}{R} \approx \frac{V_m}{V_r R f}$$

$$\frac{V_m}{V_r} = 10 \qquad R = 1000 \qquad T_P = \begin{cases} \frac{1}{60} = .0167 \\ \frac{1}{120} = .008333 \end{cases}$$

Half wave:

$$C = \frac{100}{(10)(1000)(60)} = 167\,\mu f$$

Full wave:

$$C = \frac{100}{(10)(1000)(120)} = 83.3\,\mu f$$

Normally, you will use the next larger standard value $\rightarrow$ 100 μf for full wave
200 or 220 μf for halfwave.

Variants of the formula --

$$C = \frac{V_m}{V_r}\,\frac{T_p}{R}$$

with current -- $V_m = I R$

$$C = \frac{I\cancel{R}}{V_r}\,\frac{T_p}{\cancel{R}} = \frac{I\,T_p}{V_r}$$

Usually, this is the easiest form to use.

or $$C = \frac{I}{V_r f}$$

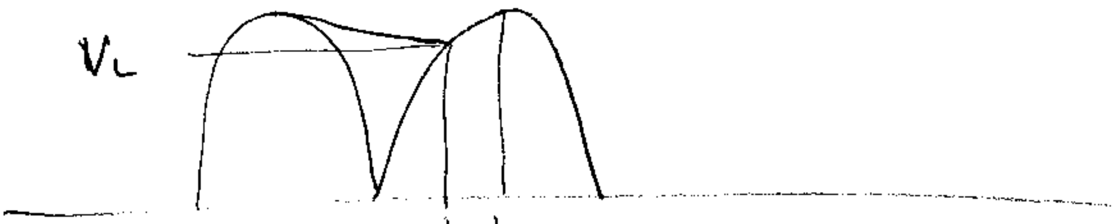
Half wave : $$C = \frac{.1}{(10)(60)} = 167 \mu f$$

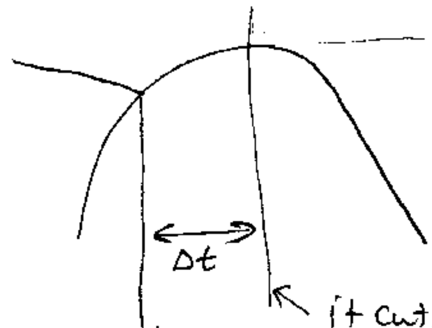
Full wave : $$C = \frac{.1}{(10)(120)} = 83.3 \mu f$$

Diode current and Capacitor current

Diode current flows only in a small part of the cycle, yet load current flows continuously. It comes from the capacitor.

Peak current is much higher than average current.





To calculate Δt ....

$$V_L = V_M \cos(\omega \Delta t)$$

Taking the first 2 terms of the taylor series –

$$\cos(x) \approx 1 - \frac{1}{2}x^2$$

$$V_L = V_M\left(1 - \frac{1}{2}(\omega\Delta t)^2\right)$$

$$V_M - V_r = V_M\left(1 - \frac{1}{2}(\omega\Delta t)^2\right)$$

$$V_M - V_r = V_M - \frac{V_M}{2}(\omega\Delta t)^2$$

$$V_r = \frac{V_M}{2}(\omega\Delta t)^2$$

$$\frac{2V_r}{V_M} = (\omega\Delta t)^2$$

example: $\sqrt{\frac{2(10)}{100}} = .44$ rad $= 25$ deg.

$$\omega\Delta t = \sqrt{\frac{2V_r}{V_M}}$$

$$\Delta t = \frac{\sqrt{\frac{2V_r}{V_M}}}{\omega}$$

Example:

$$\Delta t = \frac{\sqrt{\frac{2(10)}{100}}}{2\pi 60} = 1.18 \times 10^{-3}$$

Charge lost by capacitor to load:

$$Q = C V_r \qquad \text{also} \qquad Q = I\,T_p \quad \left(q = \frac{di}{dt}\right)$$

This charge must be restored when the diode is on..

$$Q = \underbrace{i_{cap.}}_{\text{avg. current during charge.}} \underbrace{\Delta t}_{\text{Charge time}}$$

Example:

$Q = (167\,\mu F)(10)$
$= 1.67 \times 10^{-3}$ coul.

$$i_{cap} = \frac{Q}{\Delta t}$$

Example:

$i_{cap} = \dfrac{1.67 \times 10^{-3}}{1.18 \times 10^{-3}}$

$= 1.415$ Amp.

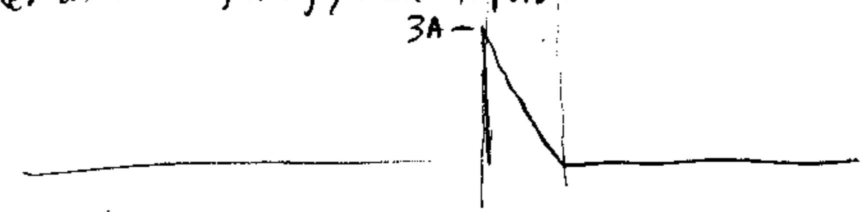
Diode current = cap current + load current

$$i_{Diode} = i_{cap} + i_{LOAD}$$

$i_D = 1.415 + .1$
$= 1.515$

But that assumes it is uniform:
It isn't.

Actual is higher at the beginning, then tapers off.



It is roughly triangular,
so the peak is about twice the average.
(3A for this example.)

Exercises!
(Not to hand in)

p	#
61	1
62-63	2, 3, 4, 5
87	4, 8, 11

To hand in:

P. 95 D 2.53

It doesn't come out exact.
Get as close as you can.
Assume $V_D = .7$
What is peak diode current?
peak inverse diode voltage?