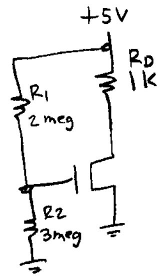
Continuing the MOSFET example

$R_1 = 2\text{meg} \quad R_2 = 3\text{meg}$

$V_{GS} = 3 \Rightarrow V_{GS} - V_{TH} = 2$



Try saturated:

$I_D = K(V_{GS} - V_{TH})^2$

$= 2(2)^2$

$= 8 \text{ ma}$

$V_{RD} = IR = (8\text{ma})(1\text{K}) = 8 \text{ volts} ???$

↑

Saturated is a Bad assumption.

Try triode:

$I_D = K\left(2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right)$

2 unknowns – need another equation ..

$V_{RD} = I_D R_D \Rightarrow I_D = \frac{V_{RD}}{R_D}$

$V_{RD} = V_{DD} - V_{DS}$

$= \frac{V_{DD} - V_{DS}}{R_D}$

substitute:

$$\frac{V_{DD} - V_{DS}}{R_D} = K\left(2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right)$$

$$\frac{V_{DD}}{R_D} - \frac{V_{DS}}{R_D} = K\left(2(V_{GS} - V_{TH})V_{DS}\right) - KV_{DS}^2$$

$$KV_{DS}^2 + \left(-2K(V_{GS} - V_{TH}) - \frac{1}{R_D}\right)V_{DS} + \frac{V_{DD}}{R_D} = 0$$

Put in values ---

$$.002\, V_{DS}^2 + \left(-2(.002)(2) - \frac{1}{1000}\right)V_{DS} + \frac{5}{1000} = 0$$

$$.002\, V_{DS}^2 + (-.008 - .001)V_{DS} + .005 = 0$$

$$2\, V_{DS}^2 + (-9)V_{DS} + 5 = 0$$

Quadratic formula ..

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad ax^2 + bx + c = 0$$

$$V_{DS} = \frac{9 \pm \sqrt{9^2 - (4)(2)(+5)}}{2(2)}$$

$$= \frac{9 \pm \sqrt{81 - 40}}{4}$$

$$= \frac{9 \pm \sqrt{41}}{4}$$

$$= 2.25 \pm 1.601$$

because for triode region --
$(V_{GS} - V_{TH}) > V_{DS}$
$2 > 3.8$?

$V_{DS} = \begin{cases} 3.851 & \leftarrow \text{doesn't make sense} \\ .649 & \leftarrow \text{this one looks good.} \end{cases}$

$V_{RD} = 5 - .649 = 4.351$

Check:

$I_D = \frac{V_{RD}}{R_D} = \frac{4.35}{1000} = .00435\text{A} = 4.35 \text{ mA}$

$I_D = K\left(2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right)$

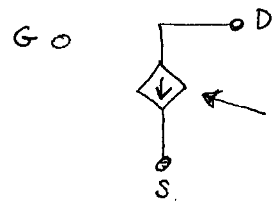
$= .002\left(2(2)(.65) - (.65)^2\right)$

$= .00435 = 4.35 \text{ mA}$

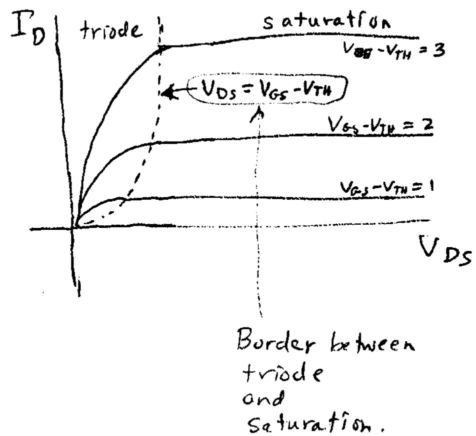
Matches = good.

Finding the gain -- Small signal model

Recall --- the large signal model:



$$I_D = \begin{cases} K(V_{GS}-V_{TH})^2 & \text{if saturation} \\ K(2(V_{GS}-V_{TH})V_{DS}-V_{DS}^2) & \text{if triode.} \end{cases}$$



For amplifier—
use saturation region.

Recall ---

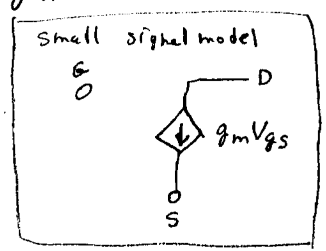
$$I_D = K(V_{GS}-V_{TH})^2$$

Want to find $\frac{dI_D}{dV_{GS}}$ = "transconductance"

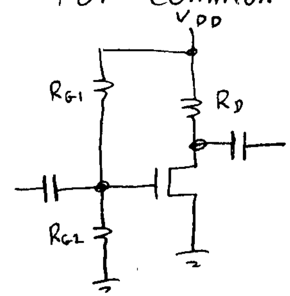
$$\frac{dI_D}{d(V_{GS}-V_{TH})} = 2K(V_{GSQ}-V_{TH}) = g_m$$

(V_{TH} ↑ constant)

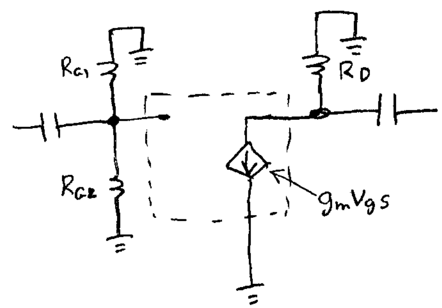
So -- $i_d = g_m v_{gs}$



For common source amplifier:

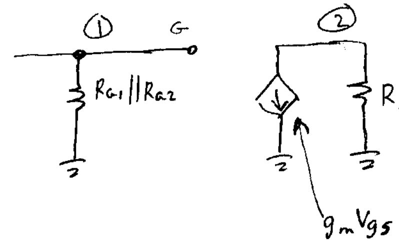


Substitute the model:



$V_{in} = V_{gs}$

↓



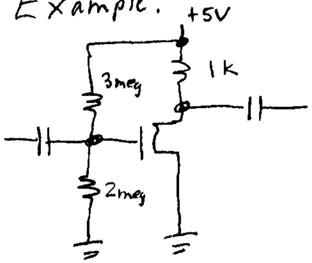
Voltage at node 2:

$V = IR$

$= g_m R_D V_{gs}$

$$\frac{V_{out}}{V_{in}} = g_m R_D$$

Example:



What is voltage gain?

First – check bias – From previous example:

$V_{GS} - V_{TH} = 1 \quad V_{DS} = 3 \quad I_D = 2ma$

Find transconductance:

$$g_m = 2K(V_{GS} - V_{TH})$$
$$= 2(.002)(1)$$
$$= .004 \ \frac{Amp}{Volt} \quad \text{or} \quad 4 \ \frac{mA}{Volt}$$

$$\frac{V_{out}}{V_{in}} = g_m R_D = (.004)(1000) = 4$$

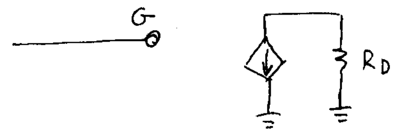
FET gain is lower than BJT gain

Note -- with BJT's you can use transconductance, too.

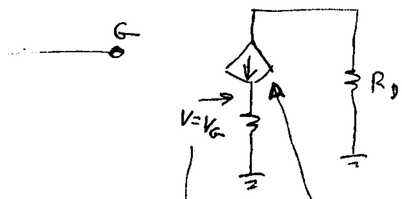
$$g_m = \frac{\beta}{r_\pi}$$

Resistor tricks –

our circuit ...



Add a resistor in series with the current source



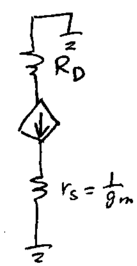
$i = g_m V_{gs}$

For voltage at this node $= V_g$

$$v = ir \Rightarrow V_{gs} = (g_m V_{gs})\, r$$

$$g_m = \frac{1}{R}$$

So – we can do:



If $g_m = .004$

then $r_s = 250\Omega$

$$\text{Gain} = -\frac{R_D}{r_s} = -\frac{1000}{250} = 4$$