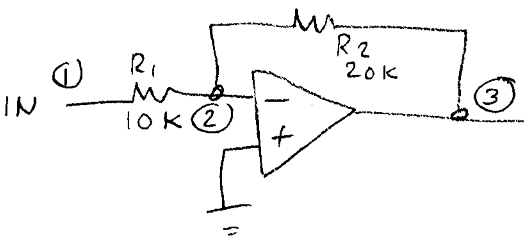
Convincing you that op-amps really work..

Recall the inverting amplifier --



closed loop.

$$\frac{V_3}{V_1} = A_v \approx -\frac{R_2}{R_1}$$

$$= -2$$

from last time.

But why does it work?
How does it amplify when there is nothing at the input?

Recall the VCVS model —

An op-amp is a VCVS, with high open loop gain $A_{OL} \approx 100\,000$.

More accurate formula for gain:

$$\frac{V_3}{V_1} = -\frac{R_2}{R_1 + \frac{R_1 + R_2}{A_{OL}}}$$

Homework —
Repeat this for $R_1 = 1K$, $R_2 = 100K$

What A_{OL} is needed for A_{CL} within 5% of ideal?

for A_{CL} = ½ of ideal?

For $R_1 = 10K$, $R_2 = 20K$ --

A_{OL}	$\frac{R_1+R_2}{A_{OL}}$	$R_1 + \frac{R_1+R_2}{A_{OL}}$	$\frac{V_3}{V_1}$
1	30K	40K	-.5
2	15K	25K	-.8
3	10K	20K	-1
4	7.5K	17.5K	-1.14
6	5K	15K	-1.33
10	3K	13K	-1.53
30	1K	11K	-1.8
100	300	10.3K	-1.94
1000	30	10.03K	-1.994
10000	3	10.003K	-1.9994
100000	.3	10,0003K	-1.99994
∞	0	10K	-2

$\frac{V_3}{V_1}$ = Closed loop gain

-2 ← Approaches $-\frac{R_2}{R_1}$

→ Closed loop gain is always lower than open loop gain.

As A_{OL} ("open loop gain") goes higher, closed loop gain approaches ideal (−2)

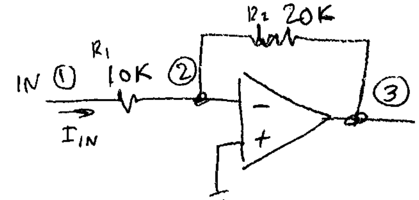
Look at the voltage at node (2).

$$V_2 = \frac{-V_3}{A_{OL}}$$

A_{OL}	V_3 (for $V_1=1$)	V_2 (for $V_1=1$)
1	−.5	.5
2	−.8	.4
3	−1	.333
4	−1.14	.285
6	−1.33	.221
10	−1.53	.153
30	−1.8	.06
100	−1.94	.0194 ≈ .02
1000	−1.994	.00194 ≈ .002
10000	−1.9994	.00019994 ≈ .0002
100000	−1.99994	.00002
∞	−2	0 ← approaches 0

We will see this idea often ---
Some tiny residual, that you approximate to 0 does all the work!

Input resistance of inverting amplifier



$R_{in} = \frac{V_{IN}}{I_{IN}}$?

By virtual short ---
If (2) is a virtual ground,
R_{in} must be $= R_1$ (10K)

For a "real" op-amp, Finite A_{OL} ---

$$I_{IN} = \frac{V_1 - V_2}{R_1}$$

$V_{IN} = V_1$

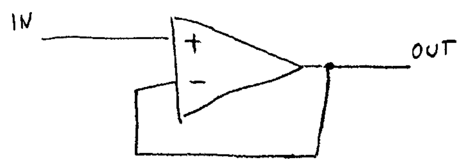
$$R_{IN} = \frac{V_1}{I_{IN}} = \frac{V_1}{\frac{V_1 - V_2}{R_1}} = \frac{V_1 R_1}{V_1 - V_2}$$

$V_2 \to 0$ as $A_{OL} \to \infty$

For $V_1 = 1$ ---

A_{OL}	V_2	$V_1 - V_2$	$\frac{V_1}{V_1 - V_2}$	R_{IN}
1	.5	.5	2	20K
2	.4	.6	1.667	16.67K
3	.333	.667	1.5	15K
4	.285	.715	1.398	13.98K
6	.221	.779	1.283	12.8K
10	.153	.847	1.18	11.8K
30	.06	.94	1.063	10.6K
100	.02	.98	1.02	10.2K
1000	.002	.998	1.002	10.02K
10000	.0002	.9998	1.0002	10.002K
100000	.00002	.99998	1.00002	10.0002K
∞	0	1	1	10K ← Approaches R_1

More op-amp circuits

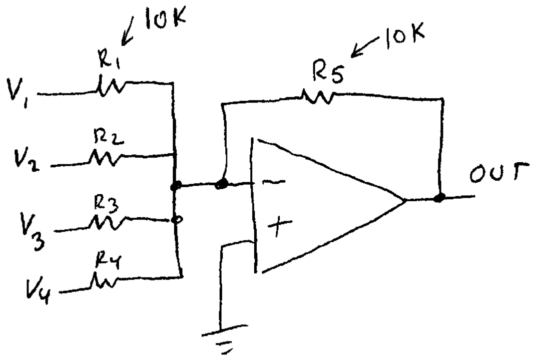


Voltage Follower.

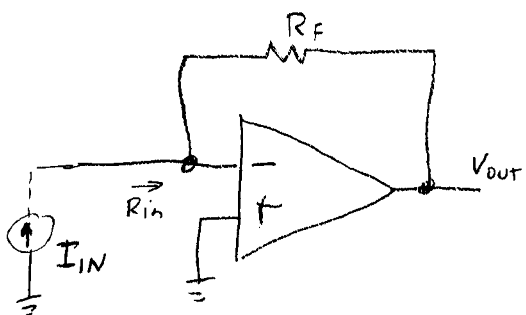
$V_{OUT} = ?$

$R_{in} = ?$

Why?



$V_{OUT} = ?$

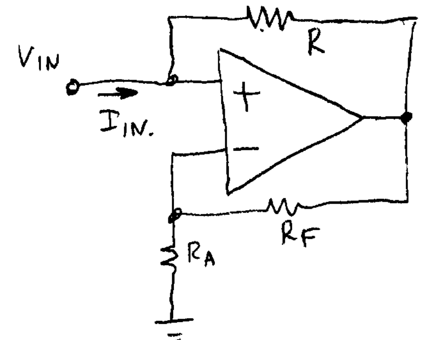


$R_{IN} = ?$

$\frac{V_{OUT}}{I_{IN}} = ?$

A strange circuit --

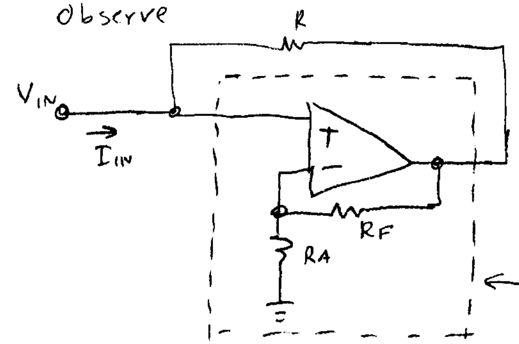
What does this do?



What is $\frac{V_{IN}}{I_{IN}}$?

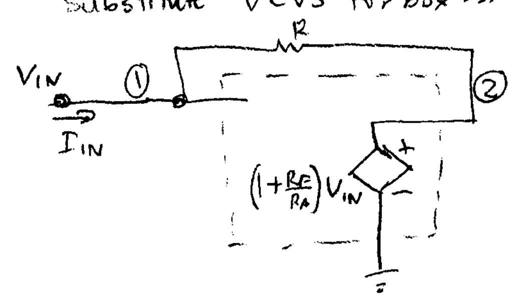
Could solve by nodal analysis — but I will cheat.

Observe



← Do you recognize this?

Substitute VCVS for box ...



$$I_{IN} = \frac{V_{IN} - V_{IN}\left(1+\frac{R_F}{R_A}\right)}{R}$$

$$= \frac{-V_{IN}\frac{R_F}{R_A}}{R}$$

$$\frac{I_{IN}}{V_{IN}} = \frac{-\frac{R_F}{R_A}}{R}$$

$$\frac{V_{IN}}{I_{IN}} = \frac{-R}{\frac{R_F}{R_A}}$$

Let $R_F = R_A$ (so gain of "box" $= 2$)

$$\frac{V_{IN}}{I_{IN}} = -R$$

Negative Resistor?

There are lots of these trick circuits that can be made with op-amps. In time, you will see more.