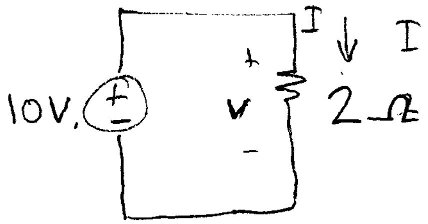
Power — real and imaginary

Review — DC power --

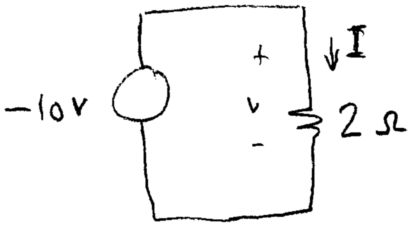
Volts × Amps.



$I = \frac{V}{R} = \frac{10}{2} = 5A$

$P_R = 10 \times 5 = 50$ watts

$P_{source} = 10 \times (-5) = -50$ watts



$I = \frac{V}{R} = \frac{-50}{2} = -5A$

$P_R = (-10) \times (-5) = 50$ watts

$P_{source} = (-10) \times (5) = -50$ watts.

Conventions:

Component polarity - assume a direction -

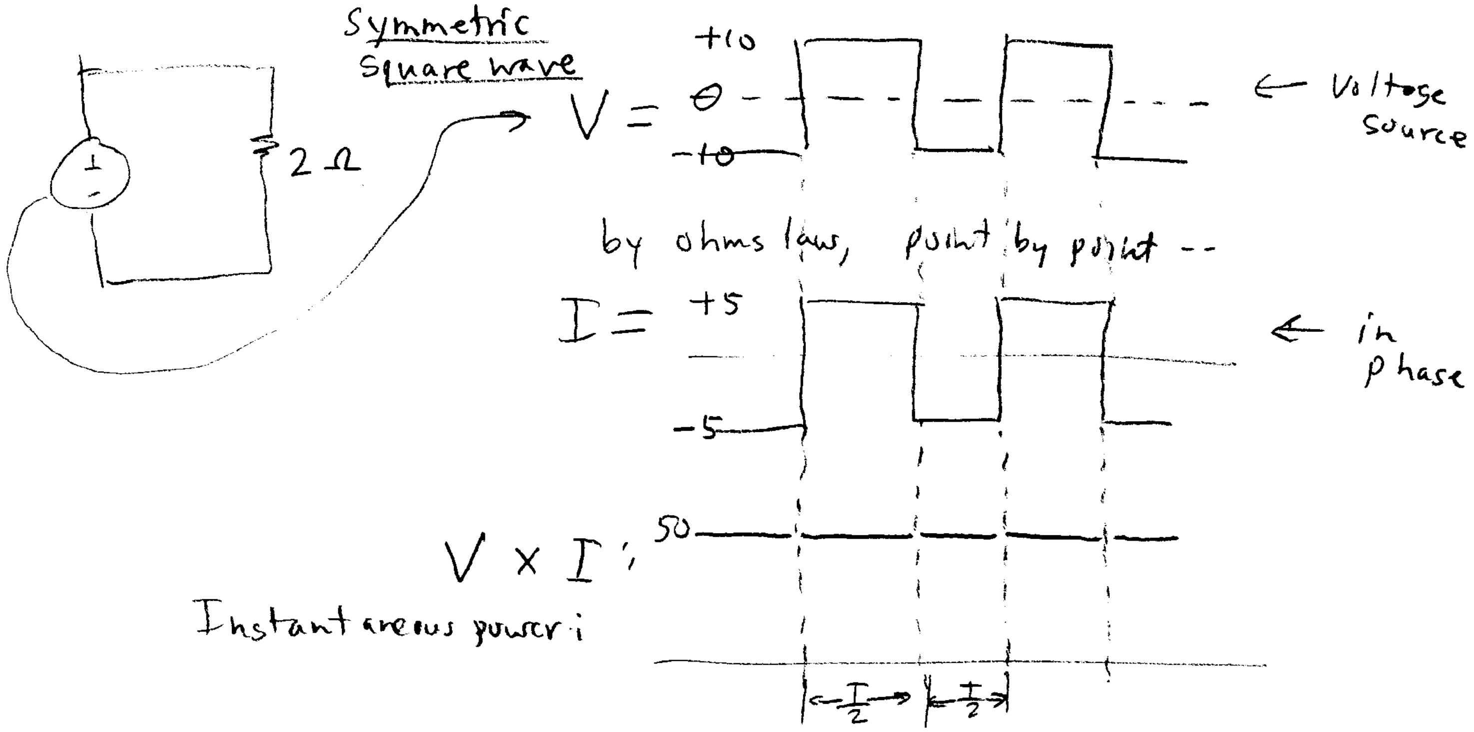
one node is +, other is -

Current flows into the $\underline{+}$ node.

power dissipated is positive

power sourced is negative

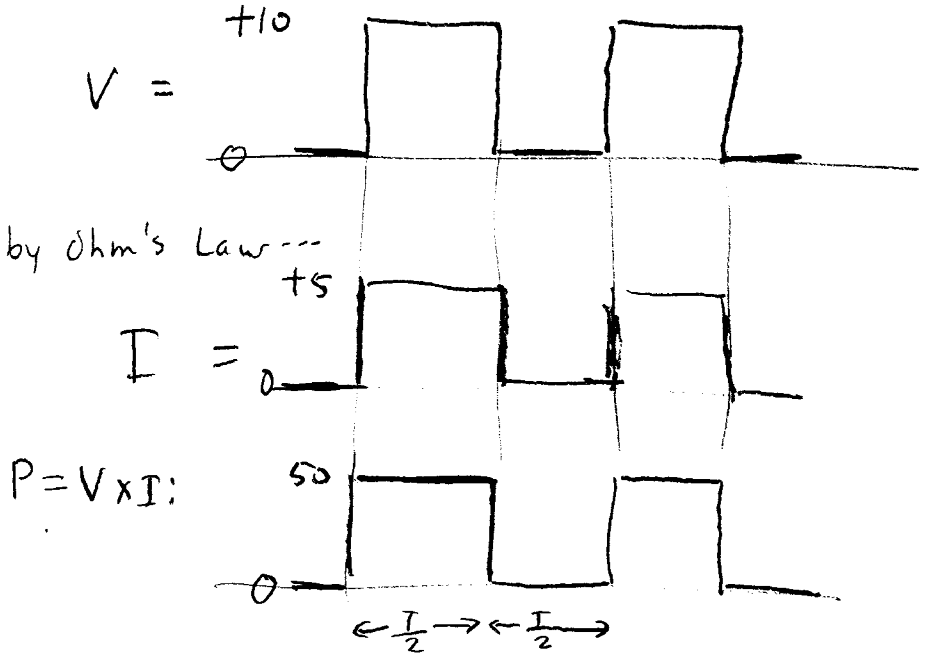
Power with time varying signals -



$$\text{Average power} = \frac{1}{T}\int_{T_0}^{T_0+T} p_i \, dt = 50 \text{ watts}$$

over a cycle.

Asymmetric square wave:



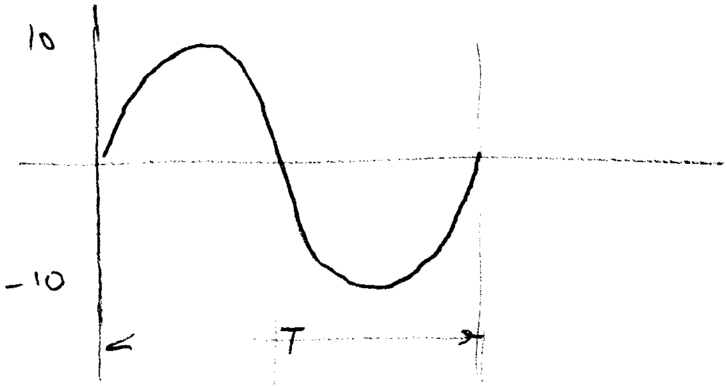
Average power --

$$P_{AV} = \frac{1}{T}\int_{T_0}^{T_0+T} p_i \, dt$$

$$\frac{1}{T}\left[\int_0^{\frac{T}{2}} 5 \, dt + \int_{\frac{T}{2}}^{T} 0 \, dt\right] = 2.5 \text{ watts.}$$

For sine wave i

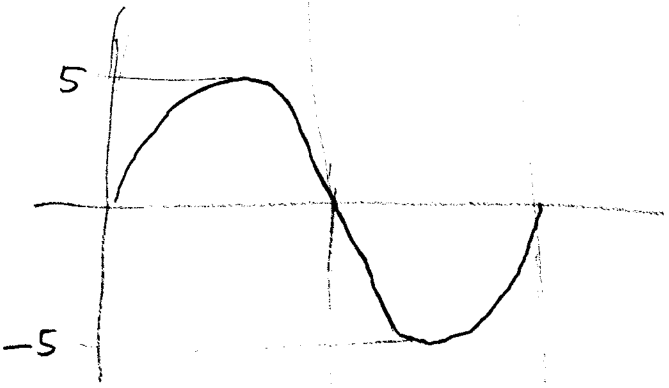
V =



$V = 10 \sin(wt)$

$= 10 \angle -90$

I =



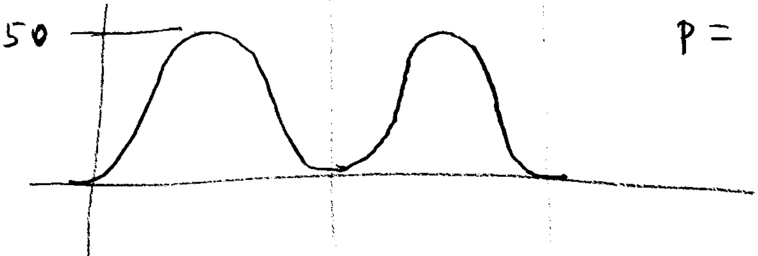
$I = \frac{V}{R}$

$= \frac{10 \sin(wt)}{2}$

$= 5 \sin(wt)$

$= 5 \angle -90$

P = V×I



$p = 10 \sin(wt) \times 5 \sin(wt)$

$= 50 \sin^2(wt)$

$= 25 - 25 \cos(2wt)$

$$P_{AV} = \frac{1}{T}\int_{T_0}^{T_0+T} P_i \, dt = \frac{1}{T}\int_{T_0}^{T_0+T} 25 + 25 \sin(2wt) \, dt$$

$$= \frac{1}{T}\left(\int 25 \, dt + \int 25 \sin(2wt) \, dt\right)$$

$$P_{AV} = \frac{1}{T}\int_{T_0}^{T_0+T} p_i \, dt =$$

$$= \frac{1}{T}\int_{T_0}^{T_0+T} 25 + 25 \sin(2\omega t) \, dt$$

$$= \frac{1}{T}\left(\int_{T_0}^{T_0+T} 25 \, dt + \underbrace{\int_{T_0}^{T_0+T} 25 \sin(2\omega t) \, dt}_{\downarrow \; = 0}\right)$$

$$= \frac{1}{T}\int_{T_0}^{T_0+T} 25 \, dt$$

$$= 25$$

Rule: For sine wave:

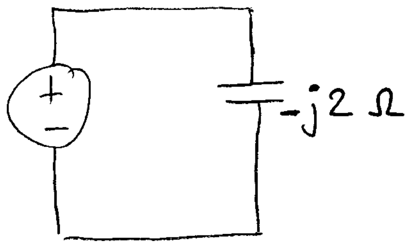
$$P = \frac{V_{peak} \times I_{peak}}{2}$$

$$= \frac{V_{peak}}{\sqrt{2}} \times \frac{I_{peak}}{\sqrt{2}}$$

$$\frac{I_{peak}}{\sqrt{2}} = I_{RMS}$$

$$\frac{V_{PEAK}}{\sqrt{2}} = V_{RMS}$$

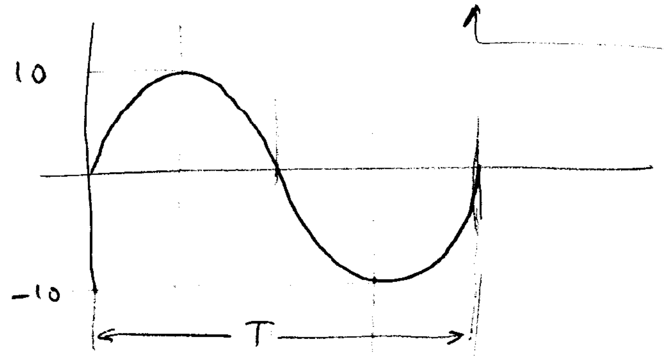
Capacitive load -- Sine wave



$V = 10 \sin(wt)$

$= 10 \angle -90°$

$V =$

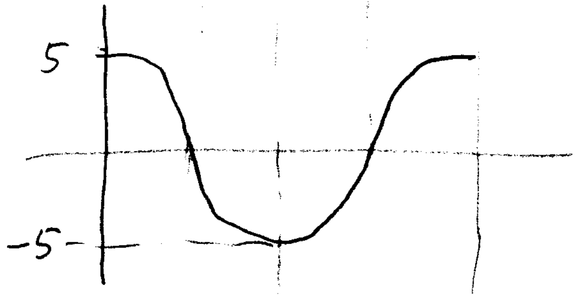


$I = \frac{V}{Z} = \frac{10 \angle -90°}{2 \angle -90°} = 5 \angle 0°$

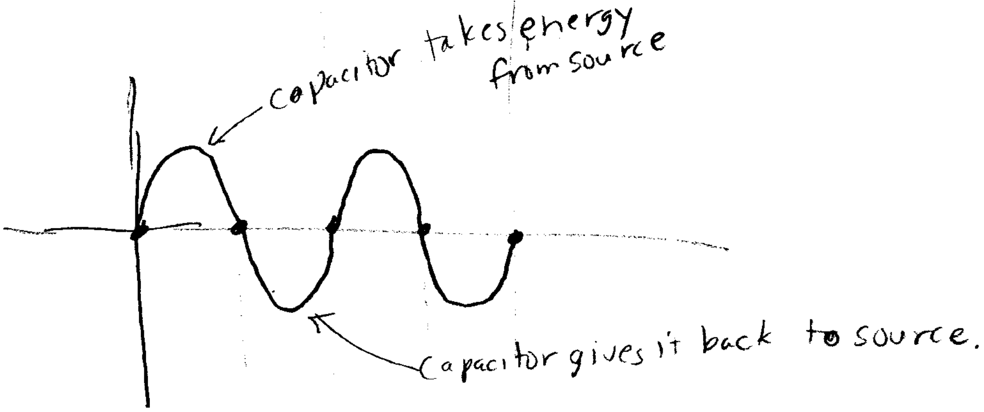
Current leads voltage by 90°

$= 5 \cos(wt)$

$I =$



$P = V \times I$



$P = 10 \sin(wt) \times 5 \cos(wt)$

$= 50 (\sin(wt) \cos(wt))$

$= 50 \times \frac{1}{2} (\sin(2wt))$

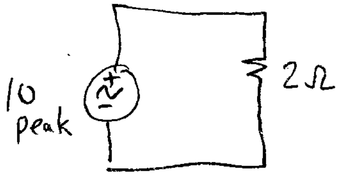
$= 25 \sin(2wt)$

$P_{AV} = \frac{1}{T} \int_{T0}^{T0+T} 25 \sin(2wt)\, dt = 0$ ← real power = 0

In general:

$$P = V_{Rms} I_{Rms} \cos(\theta)$$

↑ phase angle between voltage and current



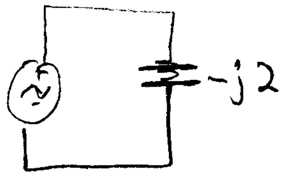
$V = 10 \sin(wt)$ $\quad V_{Rms} = 7.07$

$I = 5 \sin(wt)$ $\quad I_{Rms} = 3.54$

$\theta = 90^\circ - 90^\circ = 0$

$P = (7.07)(3.54)(\cos(0))$

$= 25$ watts.



$V = 10 \sin(wt)$ $\quad V_{Rms} = 7.07$

$I = 5 \cos(wt)$ $\quad I_{Rms} = 3.54$

$\theta = 0 - 90^\circ = -90^\circ$

$P = (7.07)(3.54)(\cos(-90^\circ))$

$= 0$ watts.

New terms:

Power factor = $\cos(\theta)$

$\uparrow$ angle between voltage and current

Real power = $V_{Rms} I_{Rms} \cos(\theta)$

The actual amount of power to do work.

Units: watts.

Apparent power = $V_{Rms} I_{Rms}$

→ Misleading number that you get if you ignore phase

Units: Volt-amps "VA"

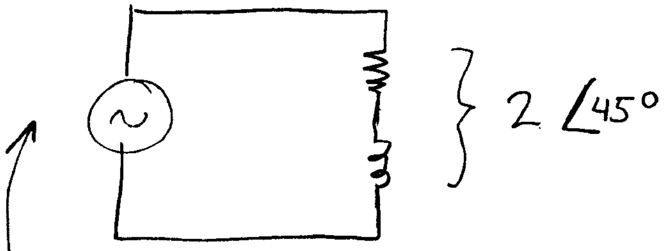
Imaginary power = $V_{Rms} I_{Rms} \sin(\theta)$

→ You think it is power, but it isn't.

It can cause a lot of trouble.

Units: voltamps reactive "VAR"

Try this one!



$V = 10 \sin(wt) = 10 \angle -90°$

$V_{Rms} = 7.07$

$I = \frac{V}{Z} = \frac{10 \angle -90°}{2 \angle 45°} = 5 \angle -135°$

$I_{Rms} = 3.54$

$\theta = (-90) - (-135) = 45°$ $\cos(\theta) = .707$

Real power

$P = V I \cos(\theta) = (7.07)(3.54)(.707)$

$= 17.67$ watts ← only real power is in watts.

Apparent power $= VI = 25$ VA ← volt-amps

Don't say watts

Imaginary power $= VI \sin(\theta)$

$= (7.07)(3.54)(.707)$

$= 17.67$ VAR ← volt-amps reactive.

Don't say watts

power factor $= \cos(\theta) = .707$