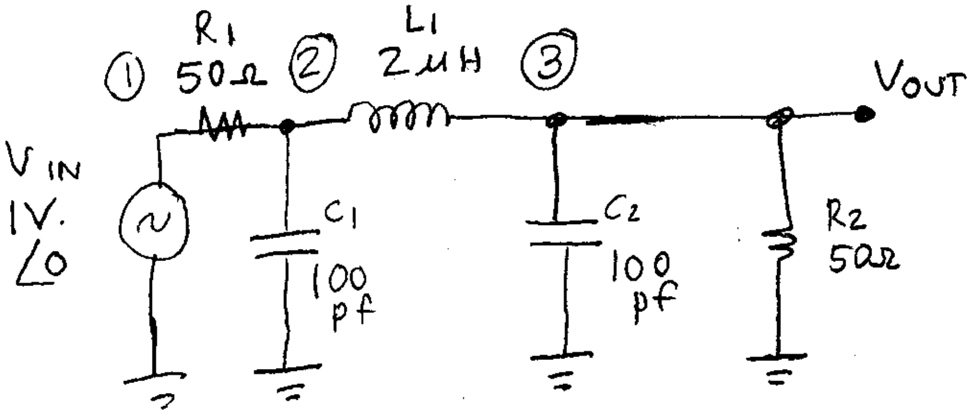
Nodal Analysis with inductors and capacitors.

It's just like resistors, except it is frequency dependent and the numbers are complex.



What is :

$$\frac{V_{OUT}}{V_{IN}} \;? \qquad \left(= \frac{V_3}{V_1}\right)$$

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22, 23, 34
36, 38

(1) $V_1 = V_{IN}$

(2) $$\frac{V_2 - V_1}{R_1} + \frac{V_2}{Z_{C1}} + \frac{V_2 - V_3}{Z_{L1}} = 0$$

(3) $$\frac{V_3 - V_2}{Z_{L1}} + \frac{V_3}{Z_{C2}} + \frac{V_3}{R_2} = 0$$

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(2) $$\frac{V_2 - V_1}{R_1} + j\omega C_1 V_2 + \frac{V_2 - V_3}{j\omega L_1} = 0$$

(3) $$\frac{V_3 - V_2}{j\omega L_1} + j\omega C_2 V_3 + \frac{V_3}{R_2} = 0$$

shorthand: let "s" = "jw"

(2) $\frac{V_2 - V_1}{R_1} + sC_1 V_2 + \frac{V_2 - V_3}{sL_1} = 0$

(3) $\frac{V_3 - V_2}{sL_1} + sC_2 V_3 + \frac{V_3}{R_2} = 0$

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(2) $V_1\left(-\frac{1}{R_1}\right) + V_2\left(\frac{1}{R_1} + sC_1 + \frac{1}{sL_1}\right) + V_3\left(-\frac{1}{sL_1}\right) = 0$

(3) $V_2\left(-\frac{1}{sL_1}\right) + V_3\left(\frac{1}{sL_1} + sC_2 + \frac{1}{R_2}\right) = 0$

Solve (3) for V2 ..

$$V_2\left(\frac{1}{sL_1}\right) = V_3\left(\frac{1}{sL_1} + sC_2 + \frac{1}{R_2}\right)$$

$$V_2 = V_3(sL_1)\left(\frac{1}{sL_1} + sC_2 + \frac{1}{R_2}\right)$$

$$= V_3\left(1 + s^2 L_1 C_2 + \frac{sL_1}{R_2}\right)$$

sub in (2)

$$V_1\left(-\frac{1}{R_1}\right) + V_3\left[\left(1 + s^2 L_1 C_2 + \frac{sL_1}{R_2}\right)\left(\frac{1}{R_1} + sC_1 + \frac{1}{sL_1}\right) - \frac{1}{sL_1}\right] = 0$$

$$V_1\left(\frac{1}{R_1}\right) = V_3\left[\left(\qquad \right)\left(\qquad \right) - \frac{1}{sL_1}\right]$$

$$\frac{V_3}{V_1} = \frac{\frac{1}{R_1}}{\left(1 + s^2 L_1 C_2 + \frac{sL_1}{R_2}\right)\left(\frac{1}{R_1} + sC_1 + \frac{1}{sL_1}\right) - \frac{1}{sL_1}}$$

We could plug in values, $s = j\omega$, here ...

If you need only one frequency ...

Example: $\omega = 10^8$

$$R_1 = R_2 = 50$$

$$Z_{C1} = Z_{C2} = \frac{1}{j\omega C} = \frac{1}{j(10^8)(10^{-10})} = \frac{1}{j10^{-2}}$$

$$Z_{L1} = j\omega L = j(10^8)(2\times10^{-6}) = j(2\times10^2) = j200$$

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(2) $$\frac{V_2 - V_1}{50} + \frac{V_2}{\frac{1}{j10^{-2}}} + \frac{V_2 - V_3}{j200} = 0$$

(3) $$\frac{V_3 - V_2}{j200} + \frac{V_3}{\frac{1}{j10^{-2}}} + \frac{V_3}{50} = 0$$

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(2) $$\frac{V_2 - V_1}{50} + \frac{V_2}{-j100} + \frac{V_2 - V_3}{j200} = 0$$

(3) $$\frac{V_3 - V_2}{j200} + \frac{V_3}{-j100} + \frac{V_3}{50} = 0$$

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(2) $$4(V_2 - V_1) + \frac{2V_2}{-j} + \frac{V_2 - V_3}{j} = 0$$

(3) $$\frac{V_3 - V_2}{j} + \frac{2V_3}{-j} + 4V_3 = 0$$

$$(2)\quad 4(V_2-V_1)+j2V_2-j(V_2-V_3)=0$$

$$(3)\quad -j(V_3-V_2)+j2V_3+4V_3=0$$

$$(2)\quad V_1(-4)+V_2(4+j2-j)+V_3(j)=0$$

$$(3)\quad V_2(j)+V_3(-j+j2+4)=0$$

$$(2)\quad V_1(-4)+V_2(4+j)+V_3(j)=0$$

$$(3)\quad V_2(j)+V_3(4+j)=0$$

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solve (3) for V_2: $V_2(-j)=V_3(4+j)$

$$V_2=V_3\,\frac{4+j}{j}$$

$$=V_3(-j4-j^2)$$

$$=V_3(1-j4)$$

sub in (2)

$$V_1(-4)+V_3(1-j4)(4+j)+V_3(j)=0$$

$$V_1(-4)+V_3(4+j-j16-j^24+j)=0$$

$$V_1(-4)+V_3(4+j-j16+4+j)=0$$

$$V_1(-4)+V_3(16-j14)=0$$

$$V_1(4)=V_3(16-j14)$$

$$\frac{V_3}{V_1}=\frac{4}{16-j14}=.14159+j.12389$$

$$=.18814\angle 41.186^\circ$$