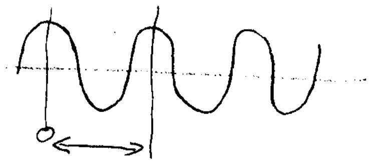
Sinusoidal Steady State Analysis (Chapter 9)

Representation : →



1 period

Frequency = "cycles per second" (Hertz).
"F"

$= \frac{1}{\text{period}}$

Hw: p.465 Ch.9.
1, 2, 3, 4, 6, 7, 8

Example: period = .001 second
Frequency = 1000 Hz.

"ω" (omega) = radians per second

$= 2\pi f$

$\omega = 2\pi 1000$ Hz

$= 6283$ rad/sec.

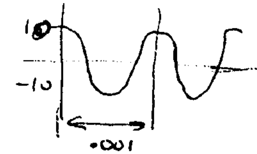
often we write it as "$2\pi f$" and don't reduce it — so the example stays "$2\pi 1000$".

$V(t) = \cos(2\pi f t)$

$= \cos(2\pi 1000 t)$ ← This example.

Amplitude:

$V_m \cos(\omega t)$
↑
peak amplitude



← $10 \cos(2\pi 1000 t)$

peak voltage = V_m

"peak-to-peak" = $2V_m$

"RMS" Voltage — the "effective value"

"root mean square"

used in computing power.

(our 10 volt sine wave contains less power than 10 volts DC)

$$V_{RMS} = \underbrace{\sqrt{}}_{\text{Root}}\!\!\!\!\!\sqrt{\underbrace{\frac{1}{T}\int_{t_0}^{t_0+T}}_{\text{Mean}} \underbrace{V_m^2 \cos^2(\omega t)\, dt}_{\text{Square}}}$$

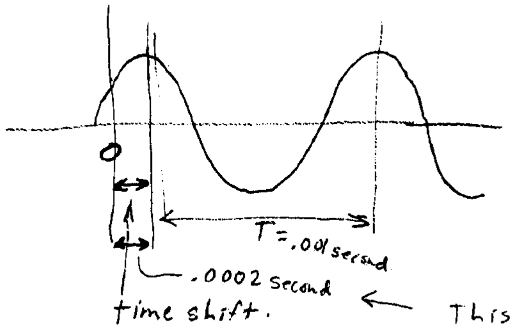
$$= \frac{V_m}{\sqrt{2}}$$

$V_m = 10$

$\Rightarrow V_{RMS} = 7.07$

Phase

A sinusoid may be shifted in time --



← This cosine wave is delayed .0002 second.

$\Delta t = -.0002$ second

Phase angle

Express the delay as an angle

in degrees: $\dfrac{-.0002}{.001} = \dfrac{\phi}{360}$

$\dfrac{\Delta t}{T} = \dfrac{\phi}{360}$

$\phi = 360 \left(\dfrac{-.0002}{.001}\right) = -72$ degrees

$\phi = 360 \dfrac{\Delta t}{T}$

$= 360\ f\ \Delta t$

$\phi = -72$ degrees

↑

negative = delayed

= shifted to right

positive = advanced

= shifted to left

in radians: $\dfrac{-.0002}{.001} = \dfrac{\phi}{2\pi}$

$\dfrac{\Delta t}{T} = \dfrac{\phi}{2\pi}$

$\phi = 2\pi \left(\dfrac{-.0002}{.001}\right) =$

$\phi = 2\pi \dfrac{\Delta t}{T} = 2\pi f \Delta t$

$= \omega \Delta t$

$= -2\pi(.2)$ radians ← often we leave it like this.

$= -1.25$ radians

Combined --

$v(t) = V_m \cos(\omega t + \phi)$

↑ radians

Be consistent --

This example

$v(t) = 10 \cos(\underbrace{2\pi\, 1000}_{\omega}\, t + \underbrace{(2\pi\, 1000)(\underbrace{-.0002}_{\Delta t})}_{\phi})$

Sometimes written:

$v(t) = 10 \cos(2\pi 1000 (t + .0002))$

$v(t) = V_m \cos(\omega(t + \Delta t))$

"Phasor" notation

There are other ways to represent this ----

Math diversion --.

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Think of cos as the real part
sin as imaginary part
of the exponential.

$$v(t) = V_m \cos(wt + \phi)$$

$$= V_m \,\mathrm{Real}\left(e^{j(wt+\phi)}\right)$$

$$= V_m \,\mathrm{Real}\left(e^{jwt} e^{j\phi}\right)$$

$$= \mathrm{Real}\left(\underbrace{V_m}_{\text{amplitude}} \underbrace{e^{j\phi}}_{\text{Phase}} \underbrace{e^{jwt}}_{\text{Frequency + time}}\right)$$

This term carries ---
Frequency + time

Phase

amplitude

For AC steady state analysis —
we use one frequency
so don't include it in the analysis —

$$V = V_m e^{j\phi}$$

big V means phasor representation

$$= V_m(\cos\phi + j\sin\phi)$$

Usually, we use a short form:

$$V_m \angle\phi$$

The example continued ---

$$v(t) = 10\cos(2\pi 1000t - 1.25\text{ rad})$$

$$V = 10\angle{-1.25\text{ rad}}$$

or $10\angle{-72^\circ}$

$$= 10(\cos(-72^\circ) + j\sin(-72^\circ))$$

$$= 10(.309 - j.951)$$

To get $v(t)$, multiply by e^{jwt}
but we won't.

7A got this far.

7B ↓

7A 7

Complex math, phasor math

Addition, subtraction —

use rectangular coordinates:

$$1\angle 0^\circ + 2\angle 45^\circ$$
$$= (1 + j0) + (1.414 + j1.414)$$
$$= 2.414 + j1.414$$
$$= 2.798\angle 30.36^\circ$$

Multiplication

Multiply the amplitudes, Add the angles. then wrap angles to ±180°

$$1\angle 0^\circ \times 2\angle 45^\circ = 2\angle 45^\circ$$
$$1\angle 45^\circ \times 2\angle 45^\circ = 2\angle 90^\circ$$
$$3\angle 70^\circ \times 2\angle 45^\circ = 6\angle 115^\circ$$
$$3\angle 110^\circ \times 2\angle 140^\circ = 6\angle 250^\circ = 6\angle -110^\circ$$

Division

Divide the amplitudes, subtract the angles then wrap to ± 180°

$$\frac{5\angle 70^\circ}{2\angle 15^\circ} = \frac{5}{2}\angle 70^\circ - 15^\circ = 2.5\angle 55^\circ$$

7A 8

The passive elements

Resistors voltage and current are in phase.

$$V = IR$$
$$\downarrow$$
$$V = R(I_{max}\cos(\omega t + \phi_i))$$

ϕ_i = phase of current

No phase change

Capacitor

$$i = C\frac{dv}{dt}$$

$$v = V_m\cos(\omega t + \phi_v)$$

phase of voltage

time domain

$$i = C\frac{d}{dt}V_m\cos(\omega t + \phi_v)$$
$$= CV_m\frac{d}{dt}\cos(\omega t + \phi_v)$$
$$= CV_m(-\omega\sin(\omega t + \phi_v))$$
$$= \omega CV_m\underbrace{(-\sin(\omega t + \phi_v))}_{\text{"j"}}$$

AC phasor

$$I = j\omega CV$$

$$V = \frac{1}{j\omega C}I = \frac{I}{\omega C}\angle -90^\circ$$

$$Z_c = \frac{1}{j\omega C}$$

voltage lags current by 90°

current leads voltage by 90°

Notation: It is common to use "s" to represent $j\omega$ --

$$Z_c = \frac{1}{sC}$$

Inductor

$V = L \frac{di}{dt}$

$i = I_m \cos(\omega t + \phi_i)$

$V = L \frac{d}{dt} I_m \cos(\omega t + \phi_i)$

$V = L I_m \frac{d}{dt} \cos(\omega t + \phi_i)$

$= L I_m (-\omega \sin(\omega t + \phi_i))$

$= \omega L I_m \underbrace{(-\sin(\omega t + \phi_i))}_{\text{"j"}}$

$V = \underbrace{j\omega L}_{} I = \omega L I \angle 90°$

$Z_L = j\omega L$
$= sL$

Voltage leads current by 90°

(current lags voltage by 90°)

Impedance $Z = \frac{V}{I}$ V and I are complex

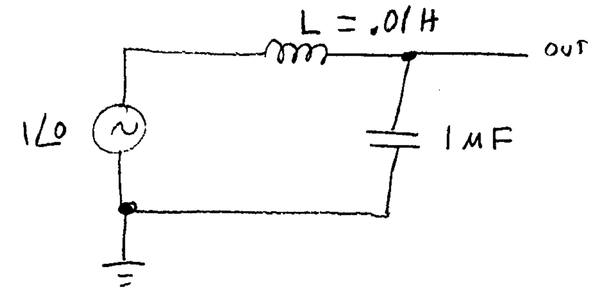
Resistance $R = \text{Real}(Z)$

Reactance $X = \text{Imag}(Z)$

$Z = R + jX$

	Z Impedance	R Resistance	X Reactance
Resistor	R	R	0
Capacitor	$\frac{1}{j\omega C}$	0	$-\frac{1}{\omega C}$
Inductor	$j\omega L$	0	ωL

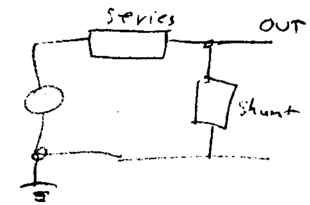
A simple circuit -- (voltage divider)



What is V_{out}?

It depends on frequency --

$V_{OUT} = V_{IN} \frac{Z_{shunt}}{Z_{series} + Z_{shunt}}$



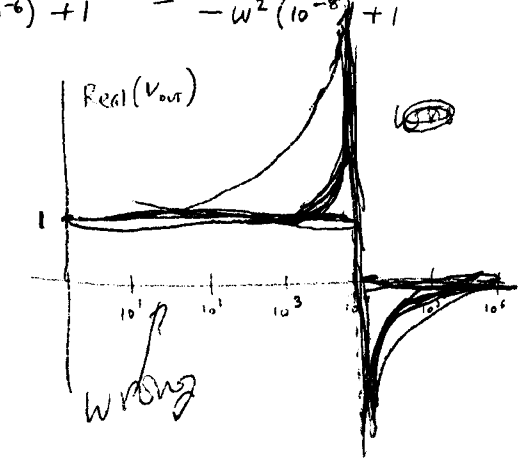
$= V_{IN} \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}$

$= V_{IN} \frac{1}{j^2\omega^2 LC + 1} = \frac{1}{-\omega^2 LC + 1}$

Sub values -

$V_{OUT} = \frac{1}{-\omega^2(.01)(10^{-6}) + 1} = \frac{1}{-\omega^2(10^{-8}) + 1}$

ω	V_{OUT}
0	1
∞	0
10^4	$\frac{1}{0}$
10^3	$\frac{1}{-10^{-2}+1}$
10^5	$\frac{1}{-10^{2}+1}$



$\frac{1}{\sqrt{LC}}$ is called "resonant frequency" (radians)

7a

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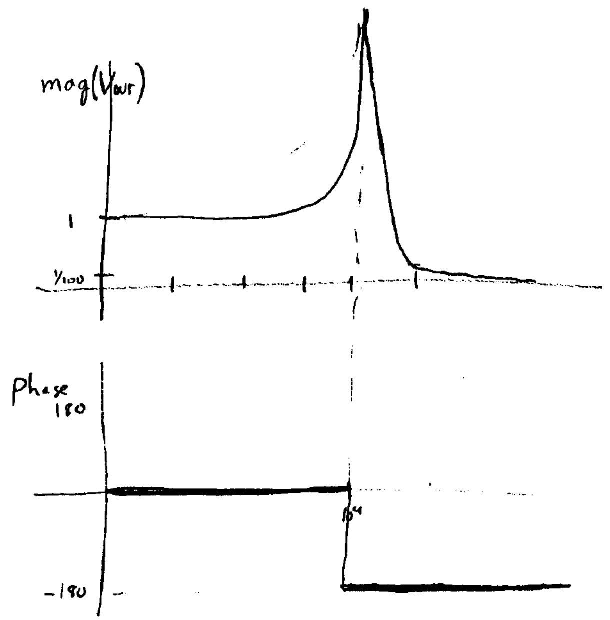

AC simulation --

The netlist is the same

commands:

print AC V(nodes) VP(nodes)

V: voltage magnitude

VP: voltage phase

print AC I(R1) IP(R1)

I: current magnitude.

IP: current phase

AC 60

60: Frequency, in Hz.

AC start stop step

AC 100 1000 100

100 Hz to 1000 Hz in 100 Hz steps.

V1 (1 0) 5 ← DC value

V1 (1 0) AC 1 ← AC value

V1 (1 0) $\underbrace{\text{DC 5 AC 1}}_{\text{both.}}$