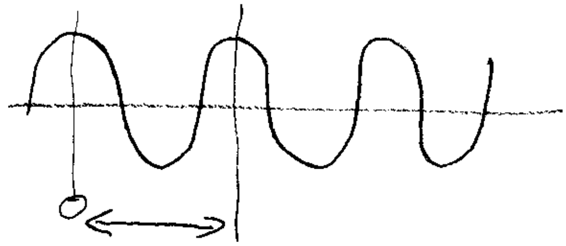
Sinusoidal Steady State Analysis (Chapter 9)

Representation: →



Frequency = "cycles per second"
"F" (Hertz).

$= \frac{1}{\text{period}}$

Example: period = .001 second
Frequency = 1000 Hz.

Hw: p. 465
Ch. 9.
1, 2, 3, 4,
6, 7, 8

"ω" = radians per second
(omega)

$= 2\pi f$

$\omega = 2\pi 1000 \text{ Hz}$

$= 6283 \text{ rad/sec.}$

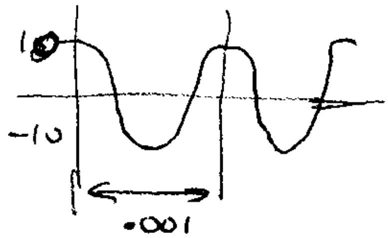
often we write it as "$2\pi f$"
and don't reduce it —
so the example stays "$2\pi 1000$".

$V(t) = \cos(2\pi f t)$

$= \cos(2\pi 1000 t)$ ← This example.

Amplitude: $V_m \cos(\omega t)$

↑ peak amplitude



$\leftarrow 10 \cos(2\pi 1000 t)$

peak voltage $= V_m$

"peak-to-peak" $= 2V_m$

"RMS" voltage — the "effective value"

"root mean square"

used in computing power.

(our 10 volt sine wave contains less power than 10 volts DC).

$$V_{RMS} = \underbrace{\sqrt{}}_{\text{Root}} \underbrace{\frac{1}{T}\int_{t_0}^{t_0+T}}_{\text{Mean}} \underbrace{V_m^2 \cos^2(\omega t)\, dt}_{\text{Square}}$$

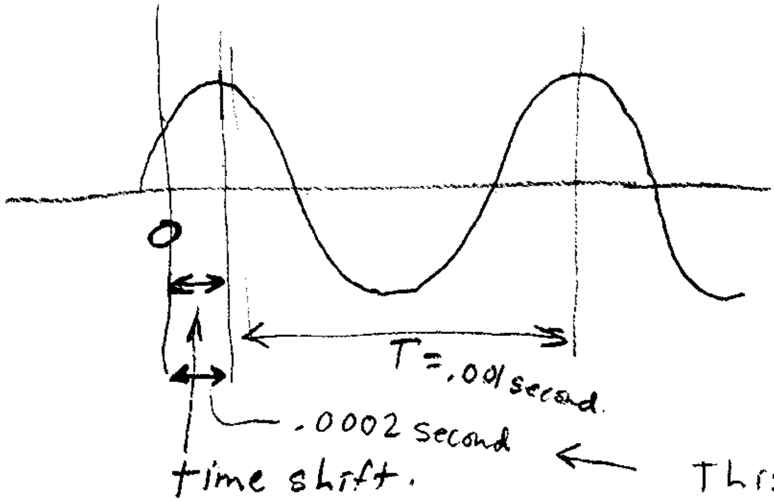
$$= \frac{V_m}{\sqrt{2}}$$

$V_m = 10$

$\Rightarrow V_{RMS} = 7.07$

Phase

A sinusoid may be shifted in time --



This cosine wave is delayed .0002 second.

$\Delta t = -.0002$ second

Phase angle

Express the delay as an angle

in degrees: $\dfrac{-.0002}{.001} = \dfrac{\phi}{360}$

$\dfrac{\Delta t}{T} = \dfrac{\phi}{360}$

$\phi = 360\left(\dfrac{-.0002}{.001}\right) = -72$ degrees

$\phi = 360 \dfrac{\Delta t}{T}$

$= 360 \, f \, \Delta t$

$\phi = -72$ degrees

negative = delayed
= shifted to right

positive = advanced
= shifted to left

in radians: $\dfrac{-.0002}{.001} = \dfrac{\phi}{2\pi}$

$\dfrac{\Delta t}{T} = \dfrac{\phi}{2\pi}$

$\phi = 2\pi\left(\dfrac{-.0002}{.001}\right) =$

$\phi = 2\pi \dfrac{\Delta t}{T} = 2\pi f \Delta t$

$= \omega \Delta t$

$= -2\pi(.2)$ radians ← often we leave it like this.

$= -1.25$ radians

Combined --

$$v(t) = V_m \cos(\underbrace{\omega t + \phi}_{\text{Be consistent}})$$

ϕ ← radians.

Be consistent . --

This example

$$v(t) = 10 \cos(\underbrace{2\pi\, 1000}_{\omega}\, t + \underbrace{(2\pi\, 1000)(.0002)}_{\phi})$$

$(.0002) = \Delta t$

Sometimes written:

$$v(t) = 10 \cos(2\pi 1000 (t + .0002))$$

$$v(t) = V_m \cos(\omega (t + \Delta t))$$

"Phasor" notation

There are other ways to represent this ----

Math diversion --

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Think of cos as the real part
sin as imaginary part
of the exponential.

$$v(t) = V_m \cos(\omega t + \phi)$$

$$= V_m \, \mathrm{Real}\left(e^{j(\omega t + \phi)}\right)$$

$$= V_m \, \mathrm{Real}\left(e^{j\omega t} e^{j\phi}\right)$$

$$= \mathrm{Real}\left(\underbrace{V_m}_{\text{amplitude}} \underbrace{e^{j\phi}}_{\text{Phase}} \underbrace{e^{j\omega t}}_{\text{Frequency + time}}\right)$$

This term carries --
Frequency + time

Phase

amplitude

For AC steady state analysis —

we use <u>one frequency</u>

so don't include it in the analysis —

$$V = V_m e^{j\phi}$$

big V means phasor representation

$$= V_m(\cos\phi + j\sin\phi)$$

Usually, we use a short form:

$$V_m \angle \phi$$

The example continued ---

$$v(t) = 10\cos(2\pi 1000t - 1.25\text{ rad})$$

$$V = 10 \angle -1.25\text{ rad}$$

or $10 \angle -72^\circ$

$$= 10(\cos(-72^\circ) + j\sin(-72^\circ))$$

$$= 10(.309 - j.951)$$

To get $v(t)$, multiply by $e^{j\omega t}$

but we won't.

Complex math, Phasor math

Addition, subtraction —

use rectangular coordinates:

$$1\angle 0^\circ + 2\angle 45^\circ$$

$$= (1 + j0) + (1.414 + j1.414)$$

$$= 2.414 + j\,1.414$$

$$= 2.798\angle 30.36^\circ$$

Multiplication

Multiply the amplitudes, Add the angles.
then wrap angles to $\pm 180^\circ$

$$1\angle 0^\circ \times 2\angle 45^\circ = 2\angle 45^\circ$$

$$1\angle 45^\circ \times 2\angle 45^\circ = 2\angle 90^\circ$$

$$3\angle 70^\circ \times 2\angle 45^\circ = 6\angle 115^\circ$$

$$3\angle 110^\circ \times 2\angle 140^\circ = 6\angle 250^\circ = 6\angle -110^\circ$$

Division

Divide the amplitudes, subtract the angles
then wrap to $\pm 180^\circ$

$$\frac{5\angle 70^\circ}{2\angle 15^\circ} = \frac{5}{2}\angle 70^\circ - 15^\circ = 2.5\angle 55^\circ$$

The passive elements

Resistors

voltage and current are in phase.

$V = IR$

$\downarrow$

$V = R\left(I_{max}\cos(\omega t + \phi_i)\right)$ — ϕ_i = phase of current

No phase change

Capacitor

$i = C\dfrac{dv}{dt}$

$v = V_m\cos(\omega t + \phi_v)$ — phase of voltage

time domain

$i = C\dfrac{d}{dt}V_m\cos(\omega t + \phi_v)$

$= CV_m\dfrac{d}{dt}\cos(\omega t + \phi_v)$

$= CV_m\left(-\omega\sin(\omega t + \phi_v)\right)$

$= \omega CV_m\underbrace{\left(-\sin(\omega t + \phi_v)\right)}_{\uparrow \text{ "j"}}$

AC phasor

$I = j\omega CV$

$V = \dfrac{1}{j\omega C}I \qquad = \dfrac{I}{\omega C}\angle -90^\circ$

$Z_c = \dfrac{1}{j\omega C}$

voltage lags current by 90°

Notation: It is common to use "s" to represent $j\omega$ -- $\qquad Z_c = \dfrac{1}{sC}$

current leads voltage by 90°

Inductor

$$V = L \frac{di}{dt}$$

$$i = I_m \cos(\omega t + \phi_i)$$

$$V = L \frac{d}{dt} I_m \cos(\omega t + \phi_i)$$

$$V = L I_m \frac{d}{dt} \cos(\omega t + \phi_i)$$

$$= L I_m (-\omega \sin(\omega t + \phi_i))$$

$$= \omega L I_m \underbrace{(-\sin(\omega t + \phi_i))}_{\text{"}j\text{"}}$$

$$V = \underbrace{j\omega L}_{Z_L = j\omega L = sL} I = \omega L I \angle 90^\circ$$

Voltage leads current by 90°

(Current lags voltage by 90°)

Impedance $Z = \frac{V}{I}$ — V and I are complex

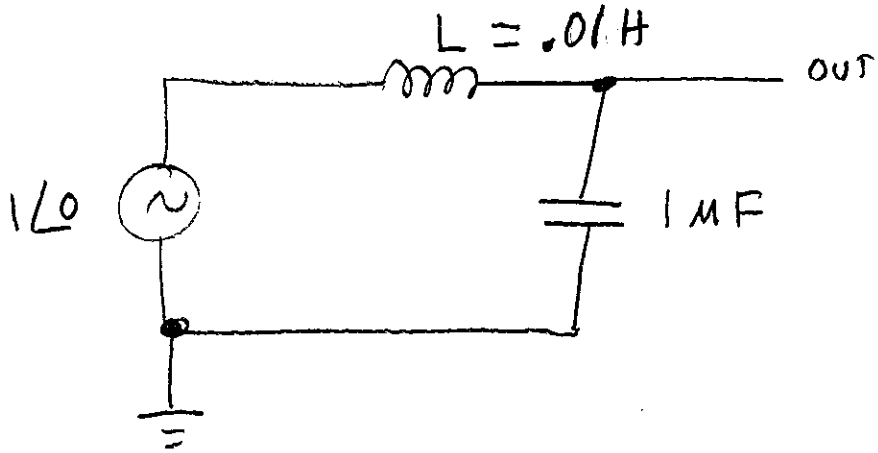
Resistance $R = \text{Real}(Z)$

Reactance $X = \text{Imag}(Z)$

$$Z = R + jX$$

	Z Impedance	R Resistance	X Reactance
Resistor	R	R	0
Capacitor	$\frac{1}{j\omega C}$	0	$-\frac{1}{\omega C}$
Inductor	$j\omega L$	0	ωL

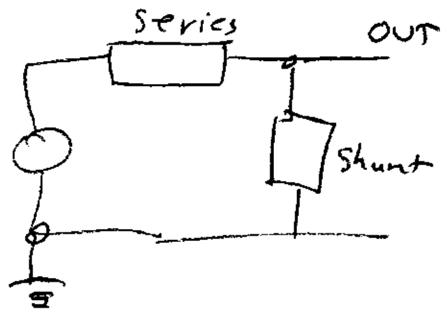
A simple circuit -- (voltage divider)



What is V_{out}?

It depends on frequency --

$$V_{OUT} = V_{IN} \frac{Z_{shunt}}{Z_{series} + Z_{shunt}}$$



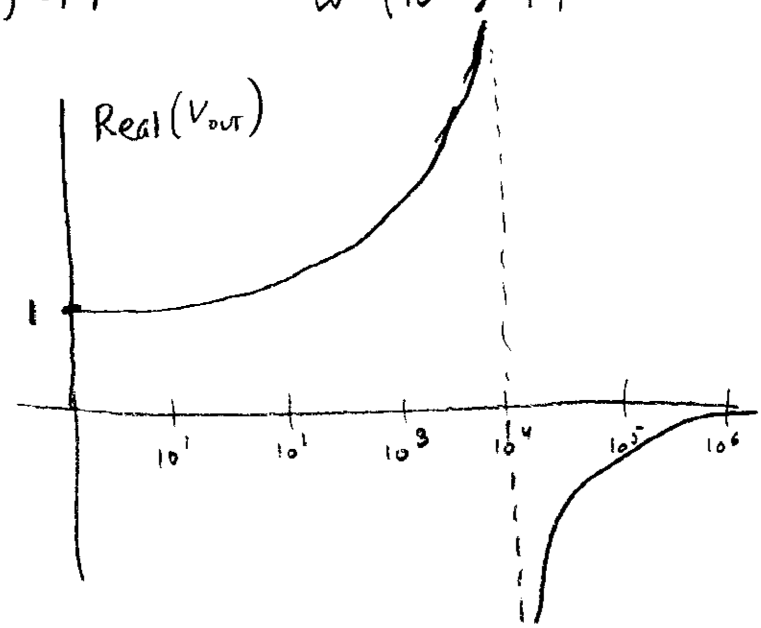
$$= V_{IN} \frac{\frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}$$

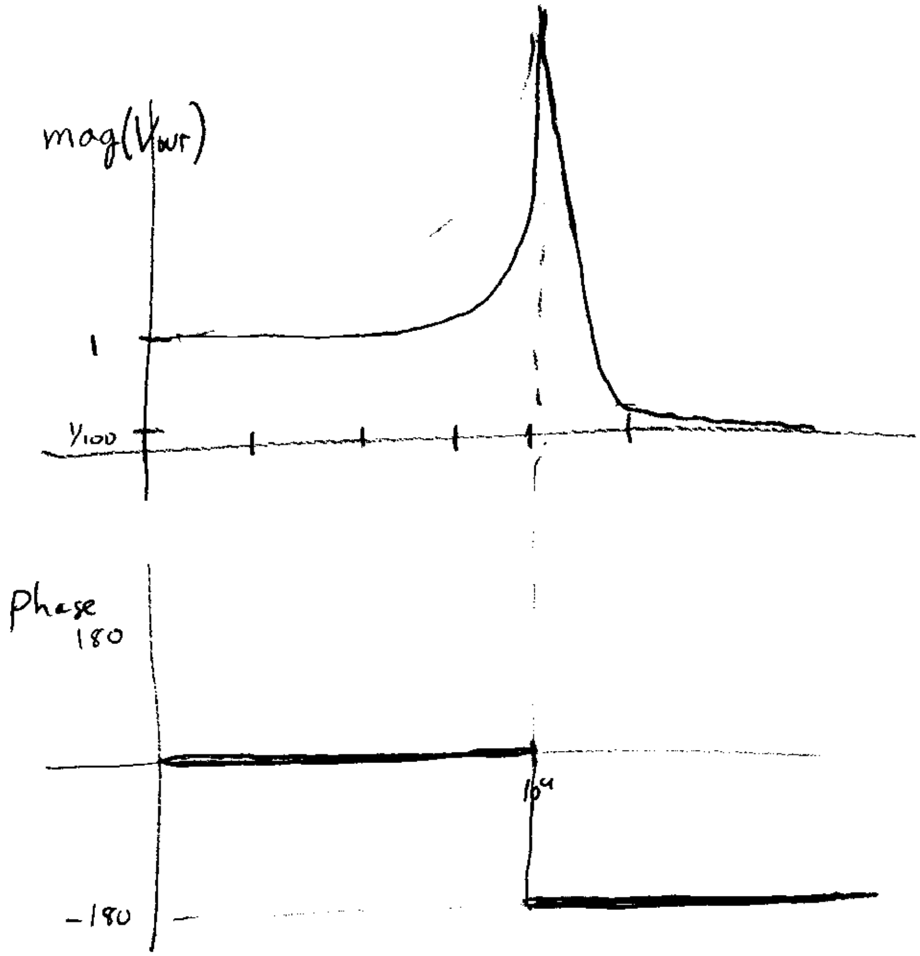
$$= V_{IN} \frac{1}{j^2\omega^2 LC + 1} = \frac{1}{-\omega^2 LC + 1}$$

sub values -

$$V_{OUT} = \frac{1}{-\omega^2(.01)(10^{-6}) + 1} = \frac{1}{-\omega^2(10^{-8}) + 1}$$

W	VOUT
0	1
∞	0
10^4	$\frac{1}{0}$
10^3	$\frac{1}{-10^{-2}+1}$
10^5	$\frac{1}{-10^{2}+1}$





AC simulation --

The netlist is the same

commands:

print AC V(nodes) VP(nodes)

V: voltage magnitude
VP: voltage phase

print AC I(R1) IP(R1)

I: current magnitude.
IP: current phase

AC 60

60: Frequency, in Hz.

AC start stop step
AC 100 1000 100
100 Hz to 1000 Hz in 100 Hz steps.