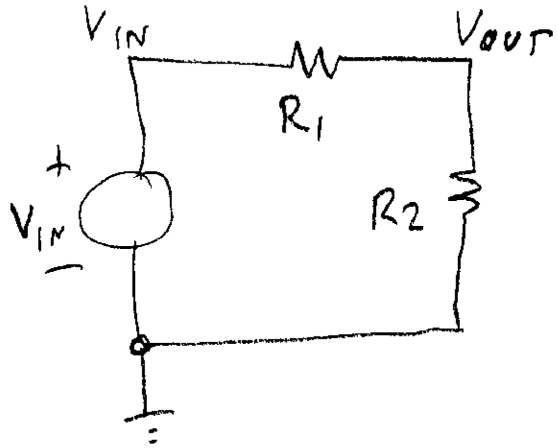
Some analysis tricks
(How to simplify a circuit)

Voltage divider



$$\frac{V_{OUT} - V_{IN}}{R_1} + \frac{V_{OUT}}{R_2} = 0$$

$$V_{OUT}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_{IN}\left(\frac{1}{R_1}\right) = 0$$

$$V_{OUT}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V_{IN}\left(\frac{1}{R_1}\right)$$

$$V_{OUT} = \frac{V_{IN}\left(\frac{1}{R_1}\right)}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= \frac{V_{IN}\left(\frac{R_1 R_2}{R_1}\right)}{\frac{R_1 R_2}{R_1} + \frac{R_1 R_2}{R_2}}$$

$$V_{OUT} = V_{IN} \frac{R_2}{R_1 + R_2}$$

HW --
Ch. 3:
10, 12
Ch. 4:
56, 58, 59, 60, 62

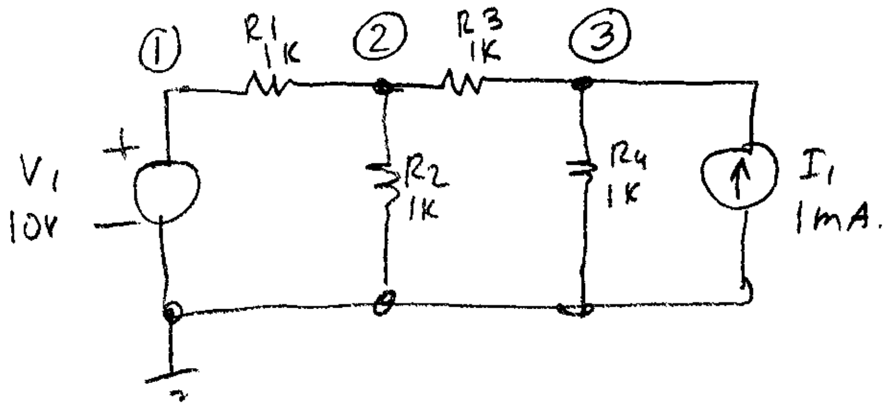
Use equivalent circuits and superposition when possible.

Analysis tricks

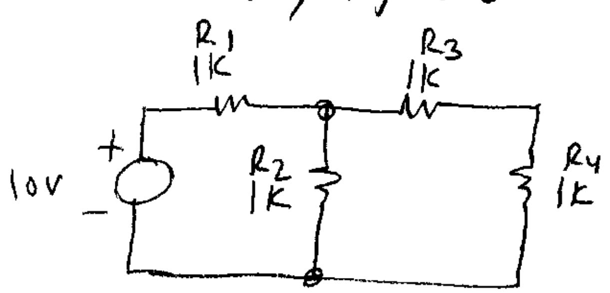
Superposition

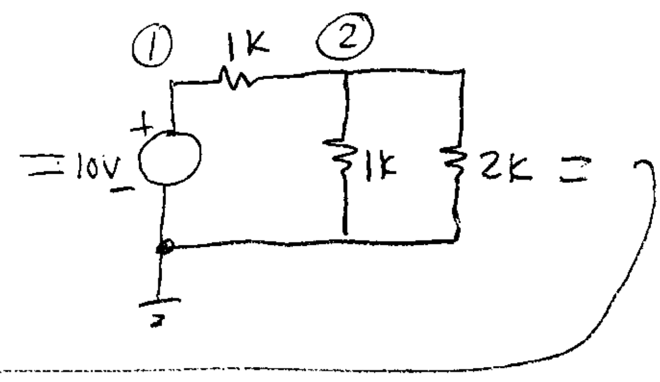
If there are multiple fixed sources, we can consider them one at a time, then add the results.

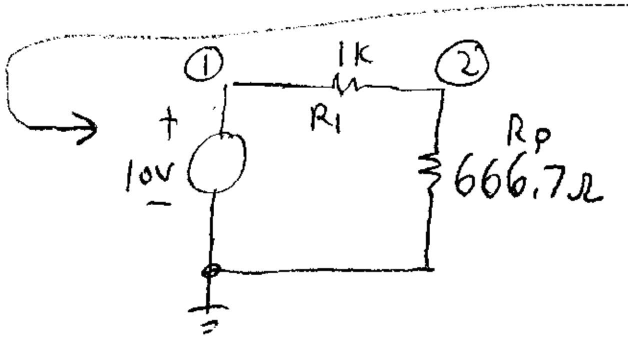
Example (Quiz question).



Consider V_1, ignore I_1 (set I_1 to 0)







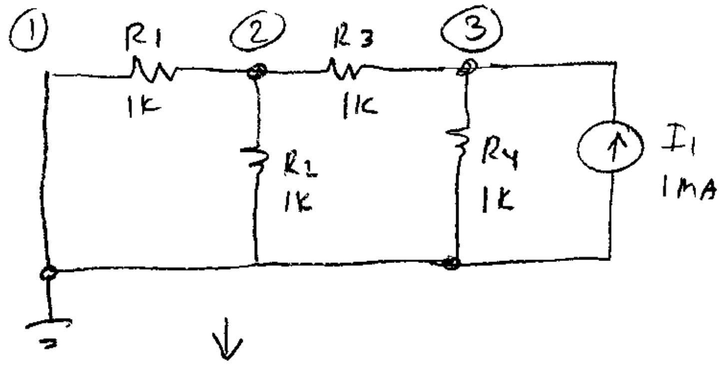
$$V_2 = V_1 \frac{R_p}{R_1 + R_p}$$

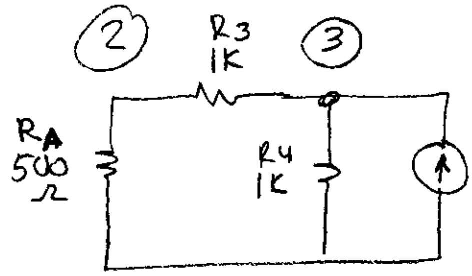
$$= 10 \frac{666.7}{1666.7}$$

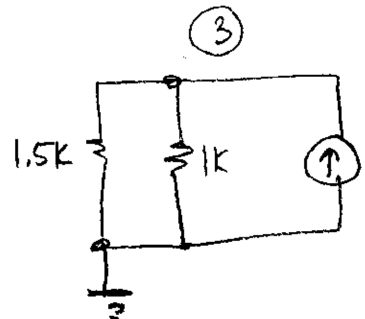
$V_2 = 4\,V.$

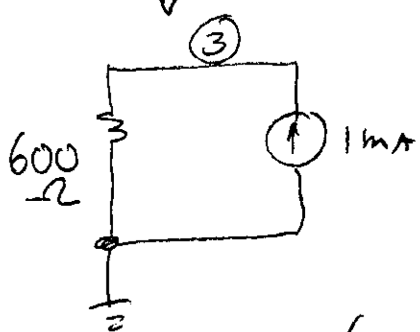
$V_3 = 2V$ ← by voltage divider

Now solve again, ignoring the voltage source.









$$V_3 = (1mA)(.6K)$$

$$V_3 = .6\,V.$$

Now solve for (2) by voltage divider.

$$V_2 = V_3 \frac{R_A}{R_A + R_3}$$

$$= .6 \frac{500}{1500}$$

$$V_2 = .2$$

Now add them:

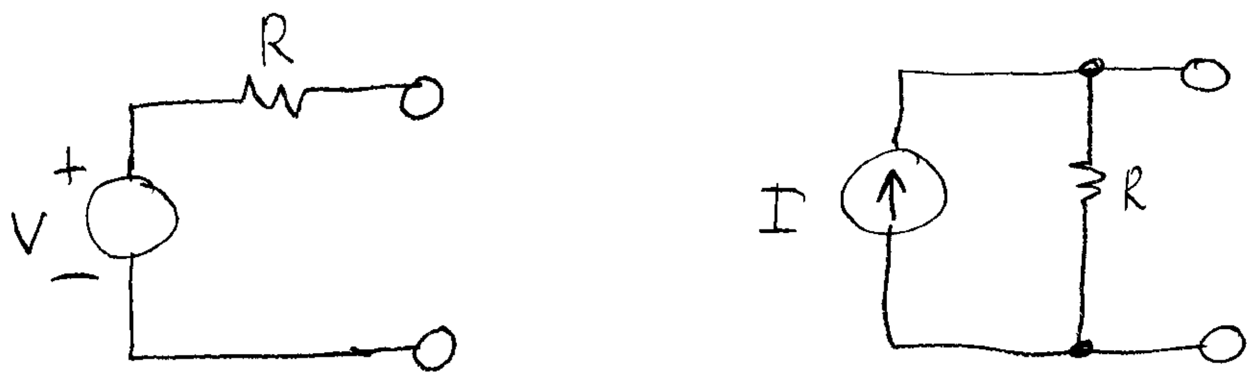
	Using only V_1	Using only I_1	Total
V_1	10	0	10
V_2	4	.2	4.2
V_3	2	.6	2.6

↑ This is the answer.

"Thevenin" and "Norton" Equivalent Circuits

↑ voltage source (Thevenin) ↑ current source (Norton)

If all we care about is what we see at the terminals — we can substitute these simple equivalents:



These are eqivalent, if V, I, R are chosen correctly

More complex networks can be replaced by these!

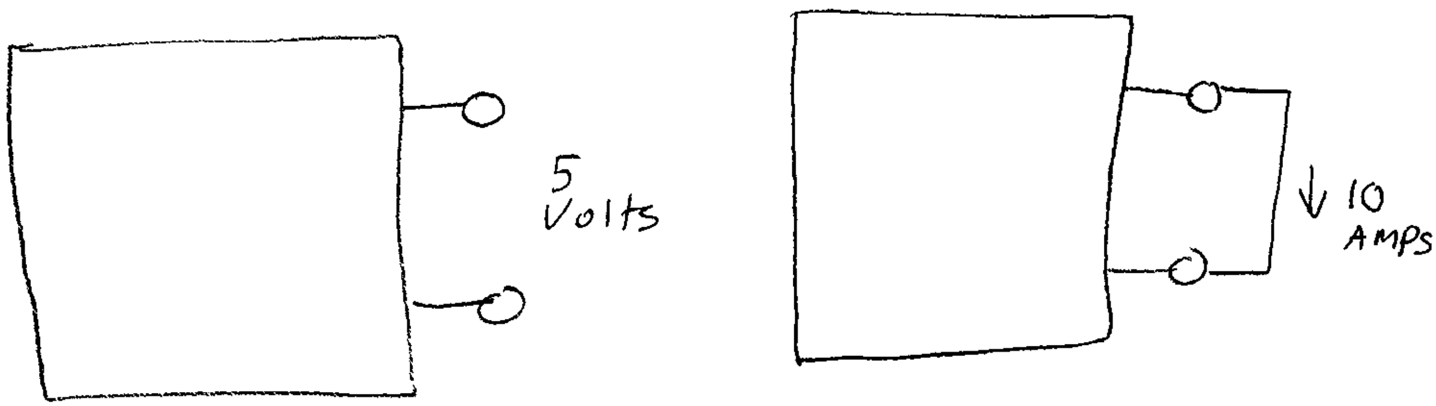
To find either:

Measure: Open circuit voltage

short circuit current

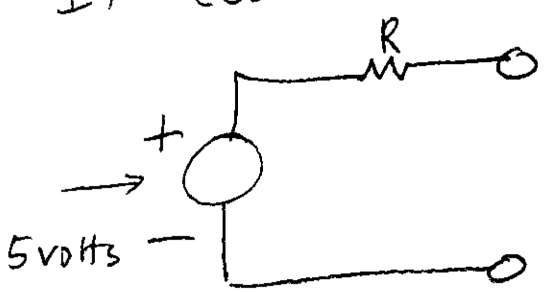
Then use ohm's law to find $R = \frac{V_{sc}}{I_{oc}}$

Example:

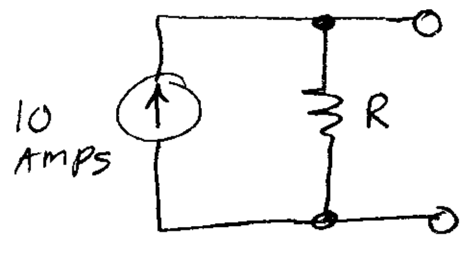


What's in the box?

It could be ----



or



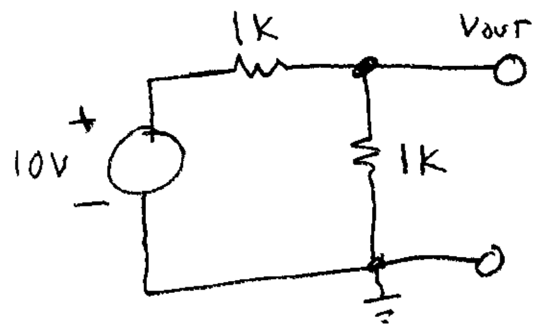
What R gives 10 Amps in short circuit?

$$R = \frac{V_{oc}}{I_{sc}} = \frac{5}{10} = .5 \text{ ohm}$$

What R gives 5 volts across open circuit?

$$R = \frac{V_{oc}}{I_{sc}} = .5 \text{ ohm}$$

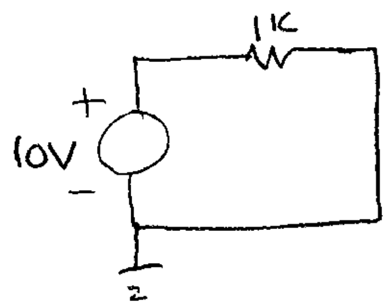
Example:



Find both equivalents.

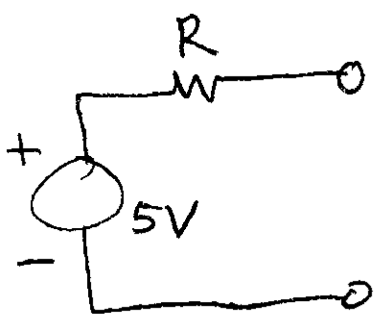
open circuit voltage: $V_{OUT} = 10 \frac{1K}{2K} = 5$

short circuit current:

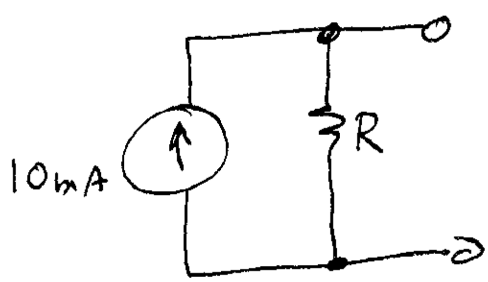


$\downarrow I = ?$ $\quad I = \frac{V}{R} = \frac{10}{1K} = 10\ mA$

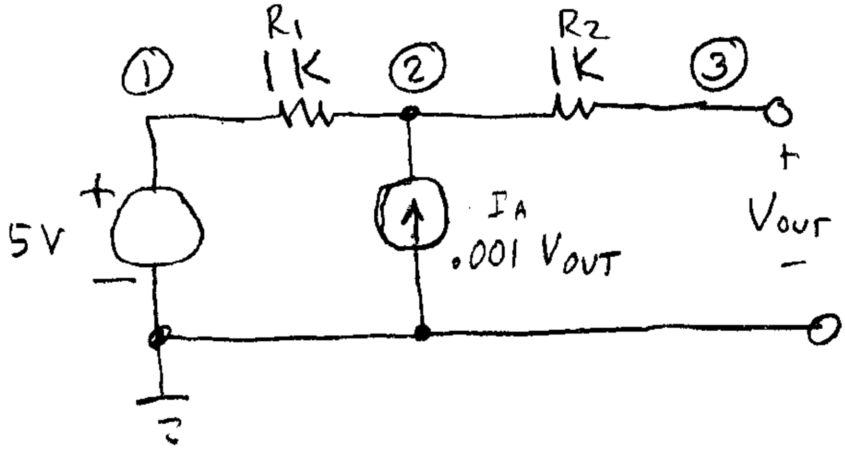
Thevenin equivalent:



Norton equivalent:



$$R = \frac{V_{oc}}{I_{sc}} = \frac{5}{.01} = 500 \text{ ohms.}$$



Find Thevenin and Norton equivalent circuits

Open circuit:

$V_3 = V_2$

$V_1 = 5$

$$\frac{V_2 - V_1}{R_1} - .001 V_2 = 0$$

$$\frac{V_2 - 5}{1K} - .001 V_2 = 0$$

$$V_2 - 5 - V_2 = 0$$

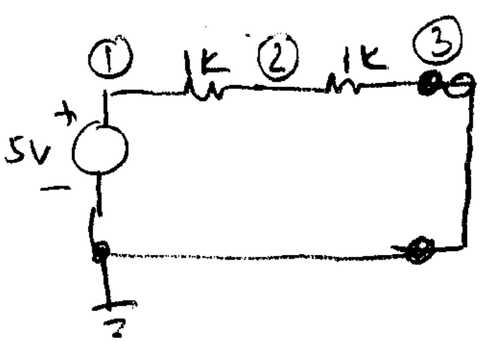
$$-5 = 0 \quad ??$$

← This circuit doesn't work!

$V_2 = \infty$

No Thevenin equivalent exists

Short circuit:



$V_3 = 0$

$I_A = 0 =$ open.

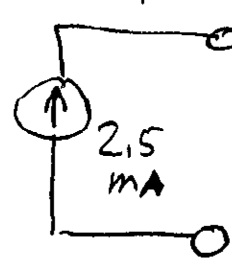
$$I = \frac{5V}{2k} = 2.5 \text{ mA.}$$

$V = \infty$

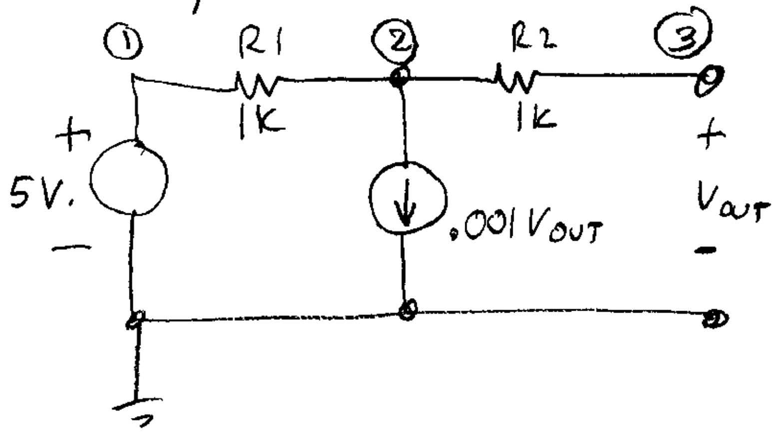
$I = 2.5 \text{mA}$

$R = \frac{\infty}{2.5\text{mA}} = \infty$

Norton equivalent:



Try this circuit!



Open circuit:

$$\frac{V_2 - V_1}{R_1} + .001\,V_2 = 0$$

$$\frac{V_2 - 5}{1K} + .001\,V_2 = 0$$

$$V_2 - 5 + V_2 = 0$$

$$2V_2 = 5$$

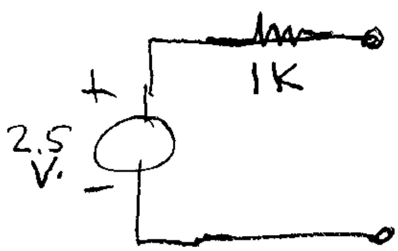
$$V_2 = 2.5$$

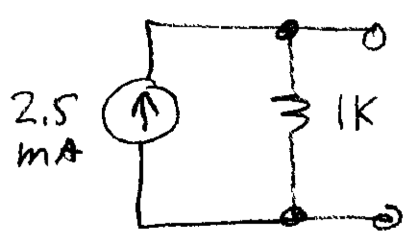
Short circuit:

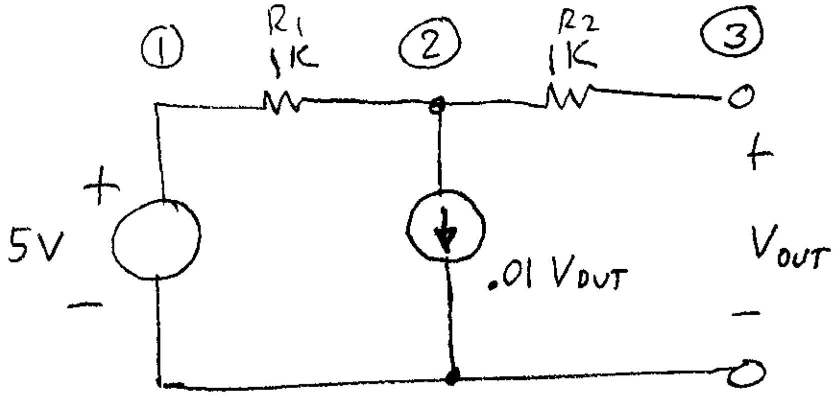
(Same as other example)

$I = 2.5$ mA.

$$R = \frac{V_{oc}}{I_{sc}} = \frac{2.5\text{ V}}{2.5\text{ mA}} = 1K$$







Open circuit:

$$\frac{V_2 - V_1}{R_1} + .01 V_2 = 0$$

$$\frac{V_2 - 5}{1K} + .01 V_2 = 0$$

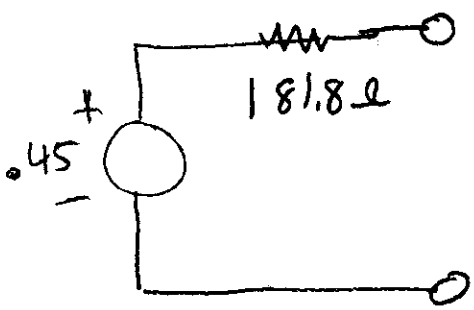
$$.001 (V_2 - 5) + .01 (V_2) = 0$$

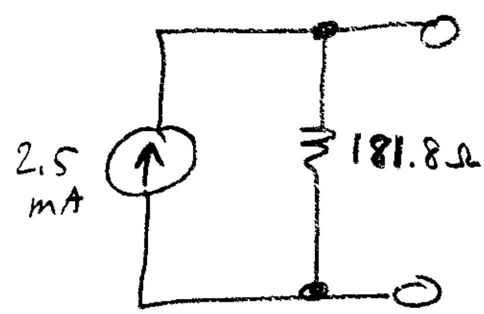
$$V_2 - 5 + 10 V_2 = 0$$

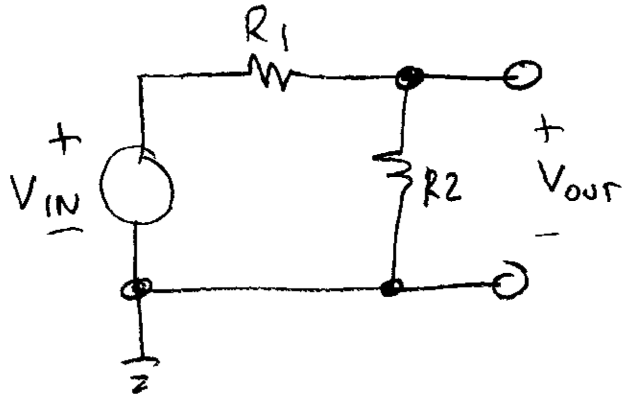
$$11 V_2 = 5$$

$$V_2 = \frac{5}{11} = V_{sc} = .45$$

$$R = \frac{V_{sc}}{I_{oc}} = \frac{.45V}{2.5mA} = 181.8\ \Omega$$







(voltage divider).

Open circuit:

$$V_{oc} = V_{OUT} = V_{IN} \frac{R_2}{R_1 + R_2}$$

Short circuit:

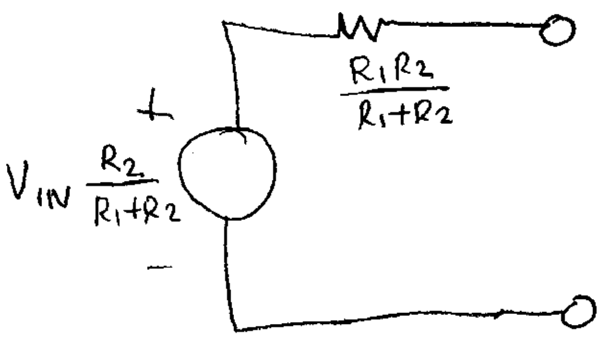
$$I_{sc} = \frac{V_{IN}}{R_1}$$

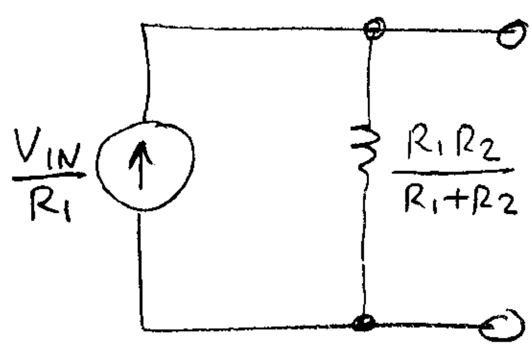
$$R = \frac{V_{oc}}{I_{sc}} = \frac{V_{IN} \frac{R_2}{R_1+R_2}}{V_{IN} \frac{1}{R_1}} = \frac{R_1 R_2}{R_1 + R_2}$$

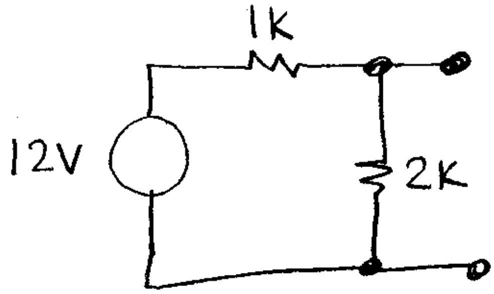
parallel combination of R_1 and R_2.

$$R = \frac{\frac{R_1 R_2}{R_1 R_2}}{\frac{R_1}{R_1 R_2} + \frac{R_2}{R_1 R_2}} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_1}}$$

← proof that these formulas are equivalent







$$V_{oc} = 12 \frac{2K}{1K+2K} = (12)\left(\frac{2}{3}\right) = 8V.$$

$$I_{sc} = \frac{12}{1K} = 12 \text{ mA}$$

$$R = \frac{8V}{12 mA} = .667 \text{ K} = 667. \ \Omega$$