

More nodal analysis

→ Floating voltage sources

The "supernode"

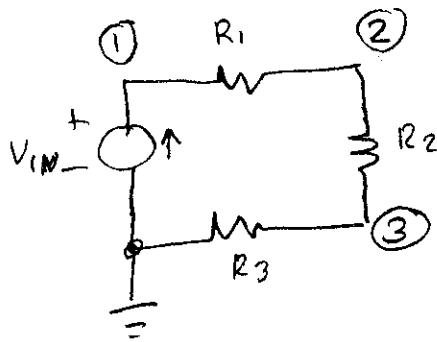
→ The "nullor" — Next week — not in text,

→ Two parts: "nullator" and "norator"

Floating Voltage sources

Problem -- Recall that we don't write a node equation for nodes connected to grounded voltage sources, because the current cannot be easily determined and the voltage is obvious.

Example:



HW
Read 4.4
Ex:
20, 25

Node ② and ③ are ordinary: ② $\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_3}{R_2} = 0$
 ③ $\frac{V_3 - V_2}{R_2} + \frac{V_3 - V_1}{R_3} = 0$

Node ① is not:

$$\textcircled{1} \quad \frac{V_1 - V_2}{R_1} - I(V_{IN}) = 0$$

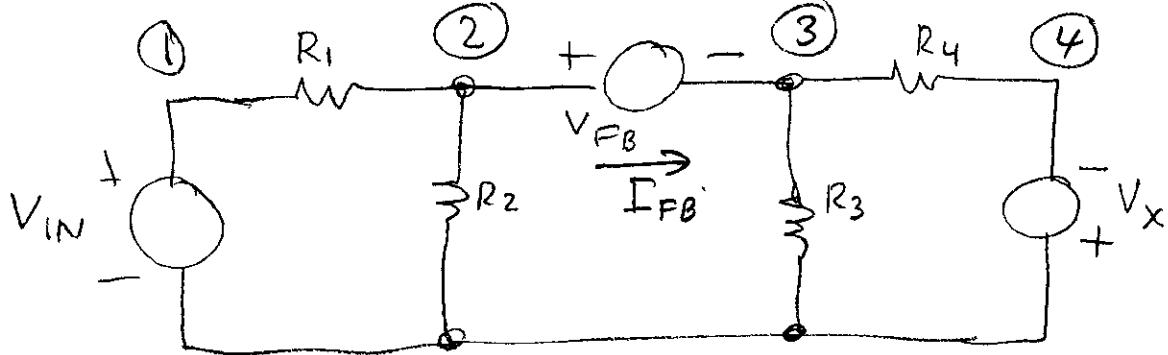
↑ what's this?

but we can just say instead:

$$\textcircled{1} \quad V_1 = V_{IN}$$

so problem solved.

But what about floating voltage sources? (2) 3C



Node ① and ④ are easy: ① $V_1 = V_{in}$
 $V_4 = -V_x$

What about ② and ③?

No easy way to calculate $I(V_{FB})$... call it I_{FB}

Just plug it in.

$$② \frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + I_{FB} = 0 \quad (\text{sum of currents out of a node} = 0)$$

$$③ \frac{V_3 - V_4}{R_4} + \frac{V_3}{R_3} - I_{FB} = 0$$

Then eliminate I_{FB} ...

$$③ I_{FB} = \frac{V_3 - V_4}{R_4} + \frac{V_3}{R_3}$$

plug into ②

$$\left[\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3 - V_4}{R_4} + \frac{V_3}{R_3} \right] = 0$$

This single equation represents both ② and ③
call it a "Supernode"

Try it with real values---

3C
3

$$\begin{array}{ll} V_{IN} = 5 & R_1 = 1K \\ V_{FB} = 2 & R_2 = 2K \\ V_x = 9 & R_3 = 3K \\ & R_4 = 4K \end{array}$$

Substitute:

$$V_1 = V_{IN} = 5$$

$$V_4 = -V_x = -9$$

$$V_2 - V_3 = V_{FB} = 2 \quad \leftarrow \text{New!} \quad \Rightarrow V_2 = V_3 + V_{FB} \\ = V_3 + 2$$

$$\frac{(V_3 + 2) - 5}{1K} + \frac{V_3 + 2}{2K} + \frac{V_3 - (-9)}{4K} + \frac{V_3}{3K} = 0$$

Solve --

Multiply both sides by 12K

$$12(V_3 - 3) + 6(V_3 + 2) + 3(V_3 + 9) + 3V_3 = 0$$

$$12V_3 - 36 + 6V_3 + 12 + 3V_3 + 27 + 4V_3 = 0$$

$$25V_3 + 3 = 0$$

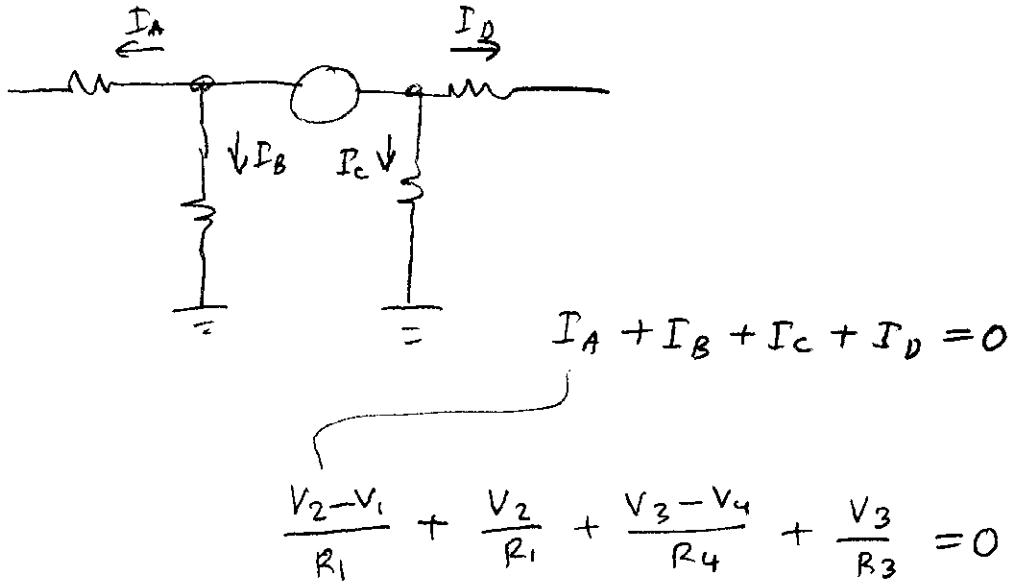
$$V_3 = -\frac{3}{25} = -0.12$$

$$V_2 = V_3 + 2 = -0.12 + 2 = 1.88$$

$V_1 = 5$
$V_2 = 1.88$
$V_3 = -0.12$
$V_4 = -9$

Observation:

We can write the supernode equation directly.
Just assume it is one node, for KCL.



and - $V_2 - V_3 = V_{FB}$

You still need 2 equations.

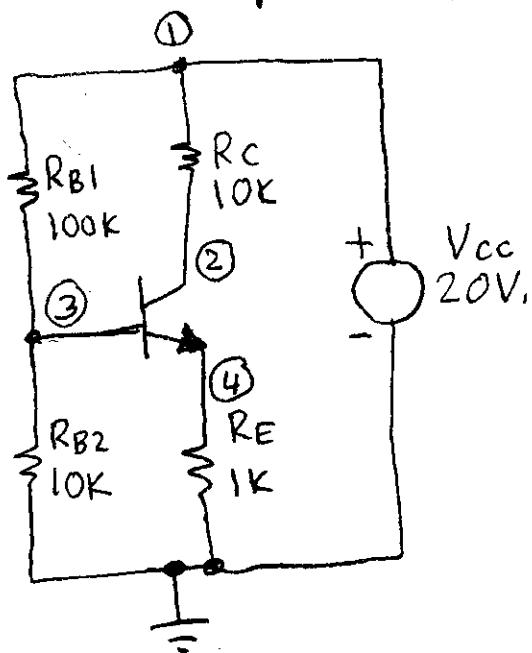
Instead of 2 KCL,

it is now 1 KCL

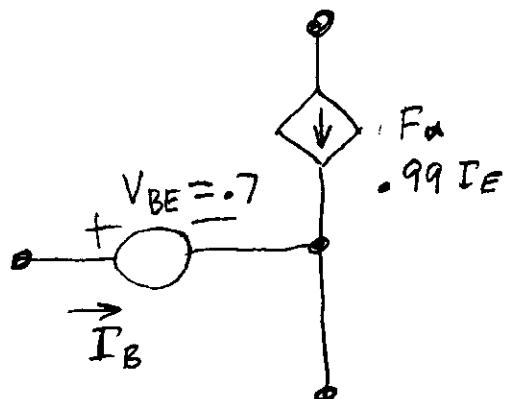
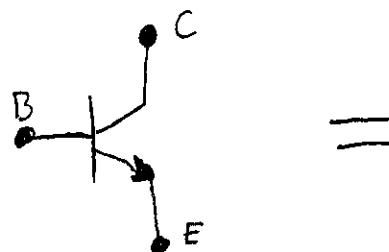
+ 1 voltage source

Supernode with a controlled source --

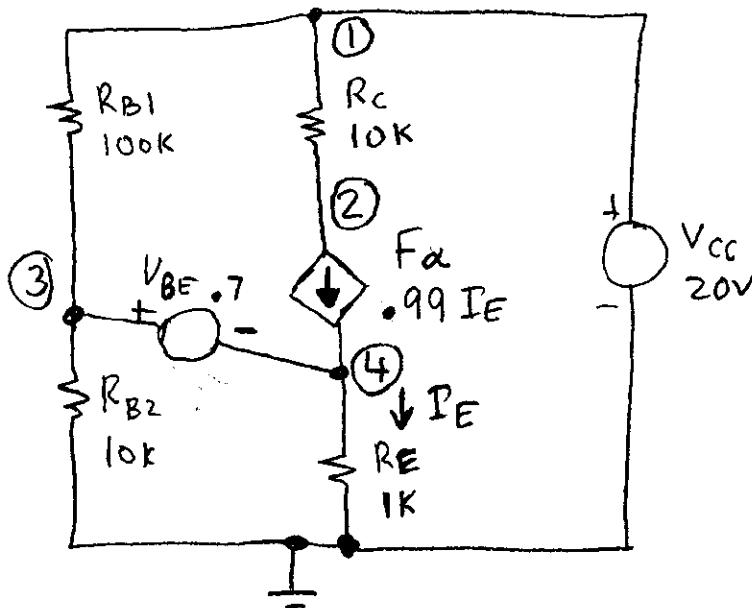
Real example from electronics:



What are node voltages?



Equivalent circuit:



$$\textcircled{1} \quad V_1 = V_{CC}$$

$$\textcircled{2} \quad \frac{V_2 - V_1}{R_C} + F_\alpha I_E = 0$$

$$\textcircled{3} \quad \left[\frac{V_3 - V_1}{R_{B1}} + \frac{V_3}{R_{B2}} + \frac{V_4}{R_E} - F_\alpha I_E \right] = 0$$

Super node

$$\textcircled{4} \quad V_{BE} = V_3 - V_4 \quad \Rightarrow \quad V_4 = V_3 - V_{BE}$$

What is I_E ?

$$V = IR \Rightarrow I = \frac{V}{R}$$

$$I_E = \frac{V_4}{R_E} = \frac{V_3 - V_{BE}}{R_E}$$

Substitute:

$$\textcircled{2} \quad \frac{V_2 - V_1}{R_C} + F_\alpha \frac{V_3 - V_{BE}}{R_E} = 0$$

$$\textcircled{3} \quad \frac{V_3 - V_1}{R_{B1}} + \frac{V_3}{R_{B2}} + \left(\frac{V_3 - V_{BE}}{R_E} - F_\alpha \frac{V_3 - V_{BE}}{R_E} \right) = 0$$

$$\textcircled{2} \quad \frac{V_2 - 20}{10K} + .99 \frac{V_3 - 7}{1K} = 0$$

$$= (1 - F_\alpha) \left(\frac{V_3 - V_{BE}}{R_E} \right)$$

$$\textcircled{3} \quad \frac{V_3 - 20}{100K} + \frac{V_3}{10K} + (1 - .99) \frac{V_3 - 7}{1K} = 0$$

Solve ③ for V_3 --

$$V_3 \left(\frac{.01}{100k} + \frac{1}{10k} + \frac{1 - .99}{1k} \right) + \left(-\frac{20}{100k} + \frac{(1 - .99)(-.7)}{1k} \right) = 0$$

$$\frac{(.01)(-.7)}{1k} = -\frac{.7}{100k}$$
3c
7

$$V_3 \left(\frac{1}{100k} + \frac{10}{100k} + \frac{1}{100k} \right) + \left(-\frac{20}{100k} - \frac{.7}{100k} \right) = 0$$

$$V_3 (12) - 20.7 = 0$$

$$V_3 = \frac{20.7}{12} = 1.725$$

$$V_4 = V_3 - .7 = 1.725 - .7 = 1.025$$

Solve for I_E :

$$I_E = \frac{V_4}{R_E} = \frac{1.025}{1k} = 1.025 \text{ mA}$$

Solve ② for V_2 :

$$\frac{V_2 - V_1}{R_C} + F_\alpha I_E = 0$$

$$\frac{V_2 - 20}{10k} + (.99)(1.025 \text{ mA}) = 0$$

$\times 10^{-3}$

mult by 10^4

$$V_2 - 20 + (.99)(10,25) = 0$$

$$V_2 - 20 + 10.15 = 0$$

$$V_2 = 9.853$$

$V_1 = 20$
$V_2 = 9.853$
$V_3 = 1.725$
$V_4 = 1.025$